A Cylindrical, non-Newtonian Liquid Jet Undergoing Linear Breakup

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An analytical model describing the breakup of a cylindrical jet of a non-Newtonian, power law fluid is developed. The long wavelength approximation is applied to the governing equations. The momentum integral approach is used to simplify those equations, which are then non-dimensionalized and then linearized to obtain a dispersion equation. Predictions for limiting cases are shown to match previously published results. The influence of non-Newtonian rheology (through the flow behavior index, n, and the consistency index, μ_n), initial liquid velocity, and surrounding air velocity on disturbance growth rate and wavenumber are then presented. Results show that an increase in either the relative velocity, the initial liquid velocity, or the surrounding gas velocity causes the jet to become more unstable (growth rate to increase). It is further observed that reducing the power law index also causes the flow to become more unstable.

1. Introduction

Atomization is necessary in many industrial processes since the liquid must be dispersed throughout the operating domain. The first step in the atomization process is usually the breakup of a flat sheet or circular jet. Breakup results from instability growth and results in drop formation. Said instability growth can be described mathematically, where the disturbance growth rate and wavenumber are predicted. It is obvious then that the wavelength for which instability-driven breakup occurs is a quantity of interest to practicing engineers since it can be used to design atomizers [1-3].

A number of theoretical and experimental investigations are available on the stability of Newtonian cylindrical jets. Rayleigh [4] was the first to develop a systematic stability theory for free surface flows and apply it to the simple case of a slow moving inviscid liquid in inertia-less surroundings. He obtained a characteristic equation to predict the range of disturbance wavenumbers that are unstable. Since the waves grow exponentially in time, the wavenumber with the maximum growth rate is expected to dominate after a small initial period of disturbance evolution. Tomotika [5] investigated the instability of a cylindrical viscous liquid thread surrounded by another viscous fluid. He applied linear stability analysis to this geometry and showed that the wavelength having the maximum growth rate is always finite when the ratio of viscosities of the two fluids is a finite value. Sterling and Sleicher [6] performed a comprehensive linear stability analysis on a cylindrical column of liquid moving in an inviscid gas.

While Newtonian liquids are common in spray applications, it is often essential to design atomization equipment for non-Newtonian fluids because these types of liquids are used in a variety of different industries, such as food processing, agricultural industry and biomedicine. Fortunately, a number of fluids being used today are found to have rheologies that obey the mathematically simple Ostwald-de-Wilde power law.

In recent times, several researchers have investigated the instability of flat sheets of non-Newtonian fluids. Ng and Mei [7] considered roll waves of mud and observed that the existence of long roll waves depends on the power-law index n. However, they did not take surface tension into account. More recently, Hwang *et al*. [8] conducted a linear stability analysis of a thin film of power-law fluid flowing down an inclined plane. Waves occurring at the surface of the film flowing down an inclined plane were also investigated analytically by Dandapat and Mukhopadhyaya [9]. All three analyses used the integral method to derive the evolution equations and then performed a linear stability analysis to determine disturbance growth rates and wavelengths. However, none of them treated the stability and breakup of non-Newtonian cylindrical jets.

The mechanisms of breakup for non-Newtonian power law cylindrical jets are interesting from both practical and theoretical standpoints. That is the reason for performing a stability analysis on such fluids. We therefore describe the evolution of a cylindrical jet with a mean velocity u_0 that is moving in a gas moving having an ambient velocity v_a. The influence of both liquid and gas velocities, along with fluid rheology, on jet breakup is described.

2. Mathematical Model

The long wavelength assumption is applied to the equations for mass conservation and momentum with the approximated governing equations being

$$\frac{\partial \mathbf{u}}{\partial r}+\frac{\mathbf{u}}{r}+\frac{\partial \mathbf{v}}{\partial z}=0 \tag{1}$$

$$r\left[\frac{\partial \mathbf{v}}{\partial t}+\mathbf{u}\frac{\partial \mathbf{v}}{\partial r}+\mathbf{v}\frac{\partial \mathbf{v}}{\partial z}\right]=-\frac{\partial p}{\partial z}-\frac{1}{r}\frac{\partial (r t_{rz})}{\partial r} \tag{2}$$

$$0=-\frac{\partial p}{\partial r} \tag{3}$$

The pressure gradient in (2) is eliminated by using (3) and the dynamic boundary condition. Integrating (1) and (2) with respect to r from 0 to h (i.e., through the sheet thickness) and using the fully developed velocity profile for a power-law fluid

$$\mathrm{v}=\frac{(\mathrm{v}_a-\mathrm{v}_0)(3n+1)}{(n+1)h_0^{\frac{n+1}{n}}}\left\{r^{\frac{n+1}{n}}-h^{\frac{n+1}{n}}\right\}+\mathrm{v}_a \tag{4}$$

yields the following averaged momentum equations

$$\frac{\partial q}{\partial z} + 2ph\frac{\partial h}{\partial t} = 0 \tag{5}$$

$$\frac{\partial q}{\partial t} - \frac{qC}{2p}h^{1/n}\frac{\partial h}{\partial z} - v_a h\frac{\partial h}{\partial t} = \frac{1}{r}\left[s\,\frac{\partial^3 h}{\partial z^3} + 2KC^n\right]\frac{h^2}{2} \tag{6}$$

Here n v_a is the velocity of the surrounding gas, n is the flow behavior index, h_0 is the initial jet radius, K is the consistency index, and

$$C = \left[\frac{1}{2K}\frac{\partial p}{\partial z}\right]^{1/n} \qquad q = \int_0^{h_0} v(2pr)dr$$

The mean velocity v_0 is defined as

$$\mathrm{v}_0 = \frac{2}{h_0^2}\int_0^{h_0} \mathrm{v}rdr = \mathrm{v}_a - 2C\frac{n}{3n+1}h_0^{\frac{n+1}{n}} \tag{7}$$

The flow field is now non-dimensionalized, decomposed into steady and perturbed components and the equations linearized

$$\frac{\partial q}{\partial z} + 2ph\frac{\partial h}{\partial t} = 0 \tag{8}$$

$$\frac{\partial q}{\partial t} + \mathrm{A}qh^{1/n}\frac{\partial h}{\partial z} - \frac{\mathrm{v}_a^*}{p}h\frac{\partial h}{\partial z} = \frac{h^2}{We}\frac{\partial^3 h}{\partial x^3} + \mathrm{A}^n\frac{h^2}{\mathrm{Re}} \tag{9}$$

where the Weber and Reynolds numbers are defined as

$$We = \frac{r\,h_0 \mathrm{v}_0^2}{s} \qquad \mathrm{Re} = \frac{r\,h_0^n \mathrm{v}_0^{2-n}}{K}$$

and

$$\mathrm{A} = \frac{(1+3n)(1-\mathrm{v}_a/\mathrm{v}_0)}{2np} \qquad \mathrm{v}_a^* = \frac{\mathrm{v}_a}{\mathrm{v}_0}$$

Equations (8) and (9) are combined to obtain the following

$$\frac{\partial^2 h}{\partial t^2} + \frac{1}{2p}\frac{\partial^2 h}{\partial z^2}\left[\mathrm{v}_a^* - A(1+q)(1+h/n)\right] + \frac{1}{2pWe}\frac{\partial^4 h}{\partial z^4} + \frac{1}{p\,\mathrm{Re}}e^{-ip\,n}\mathrm{A}^n\frac{\partial h}{\partial z} = 0 \tag{10}$$

Equation (10) is linearized and a traveling wave perturbation inserted to obtain the dispersion equation

$$w^2 - \frac{k^2}{2p}(\mathrm{v}_a^* - \mathrm{A}) + \frac{k^4}{2pWe} + \frac{k}{p\,\mathrm{Re}}\left[sin(np) + icos(np)\right] = 0 \tag{11}$$

The dispersion equation given by Equation (11) is solved for the maximum growth rate. The wavelength corresponding to the maximum growth rate is directly related to the drop size via mass conservation. Results are presented for varying operating and physical parameters.

The model developed above was validated against previously reported predictions. Substituting boundary conditions used by Sterling and Sleicher [6], namely an inviscid

Newtonian cylindrical jet, we obtained the same value of maximum growth rate that they reported.

3. Results and Discussion

Model predictions are plotted with the growth rate (ω) on the y-axis and the wavenumber (k) on the x-axis. The effect of varying mean liquid (v_0) and ambient (v_a) velocities on the breakup process was investigated first with results shown in Figs. 1 and 2.

As expected, an increase in mean liquid velocity from 25 to 45 m/s at a fixed surrounding air velocity of 15 m/s leads to increases in both growth rate and the wavenumber for which the growth rate is a maximum. See Fig. 1. The latter observation implies that drop size will decrease. In addition, the range of disturbance wavenumbers that can lead to breakup increases as the initial liquid velocity increases.

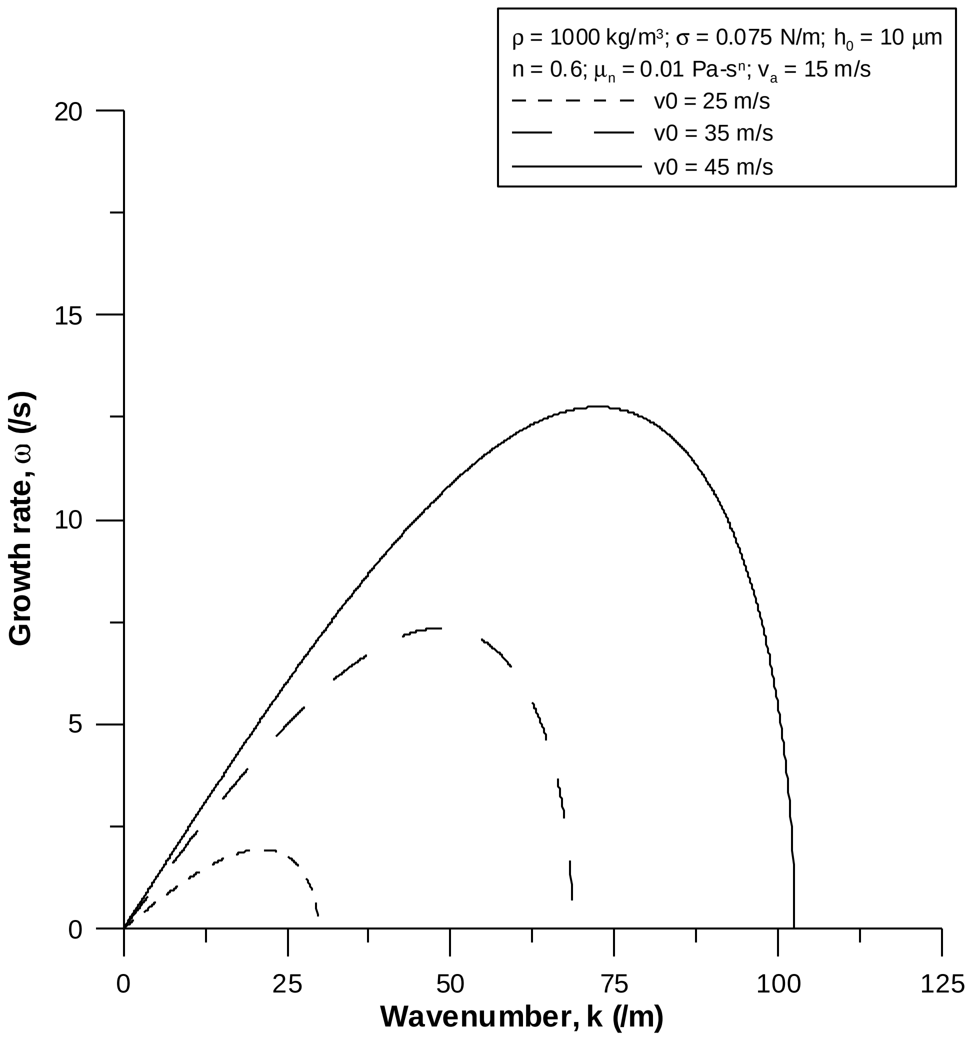

Figure 1. Growth rate (ω) versus wavenumber (k) for varying mean liquid velocities (v_0) and n = 0.6.

Fig. 2 shows that surrounding air velocity also plays a key role in breakup. In this figure, we observe that increasing the surrounding air velocity from 5 to 25 m/s while keeping the initial liquid velocity constant (at a value of 35 m/s) leads to a decrease in the growth rate and the wavenumber at which the maximum growth rate occurs. This indicates that drop size will increase. This decrease in relative velocity results in a reduction in the convective term, as shown in Eqn. (10). Since the convective term promotes breakup, its reduction leads to attenuation of the breakup process. A corresponding decrease in the wavenumber at maximum growth rate also occurs, thus leading to larger drops. Finally, Fig. 2 also shows that the range of wavenumbers that can lead to drop breakup decreases as the surrounding gas velocity approaches the initial liquid velocity.

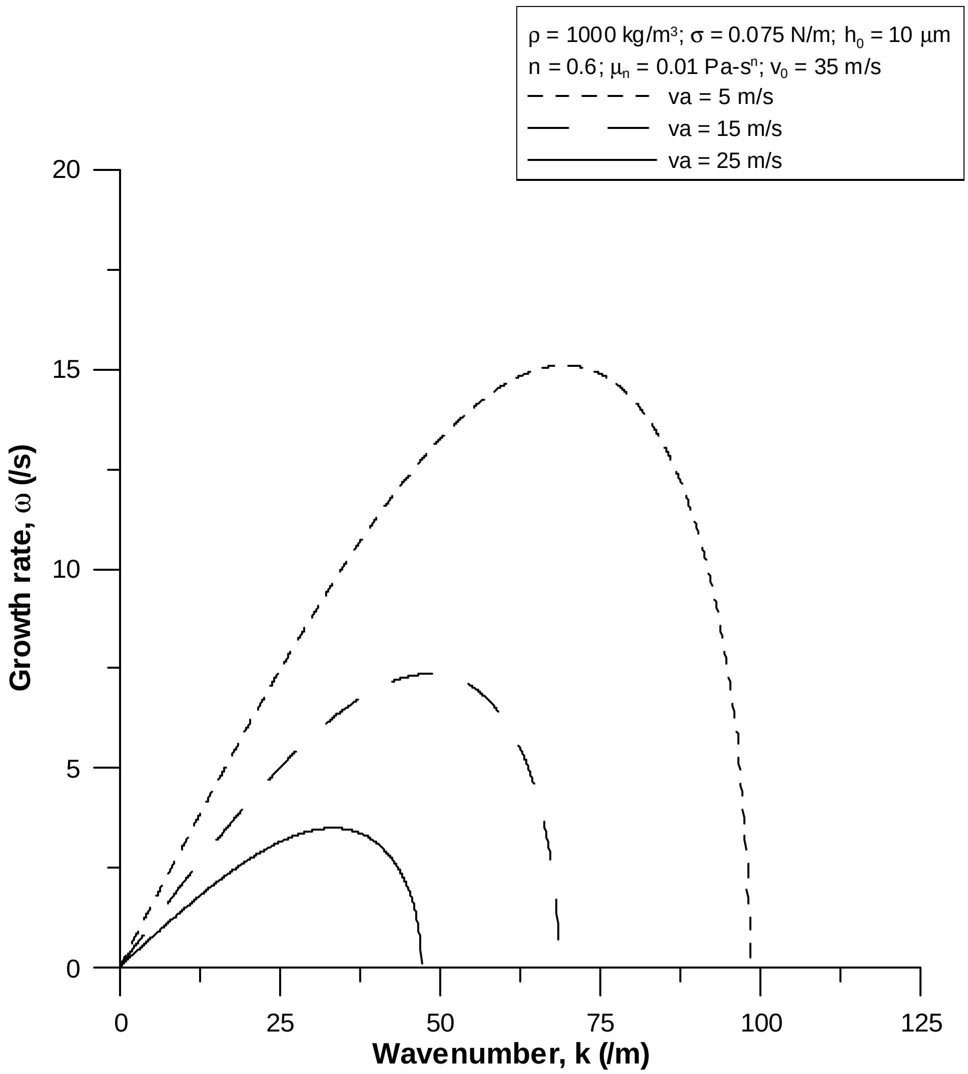

Figure 2. Growth rate (ω) vs. wavenumber (k) for varying ambient velocities (v_a) with v_0 = 35 m/s.

A comparison of Figs. 1 and 2 data help explain the effect of the varying mean velocity on breakup. It is seen from Eqn. (10) that as mean velocity increases the viscous term (which opposes breakup) decreases while the convective term (which enhances breakup) increases. Thus, an increase in mean velocity causes the maximum growth rate to increase, resulting in smaller drops. It may be noted that the relative velocity appears in both the convective part as well as the viscous part of the dispersion equation. However its affect on the viscous term is smaller than its affect on the convective term, and thus, an increase in the relative velocity leads to an overall increase in the growth rate and to decreased drop sizes.

Fig. 3 shows breakup growth rate versus disturbance wavenumber behavior for relative velocities of 20 and 30 m/s, where the mean liquid velocity was 35, 45 or 55 m/s. In each case the larger growth rates are for v_{rel} = 30 m/s while the lower growth rates are for v_{rel} = 10 m/s.

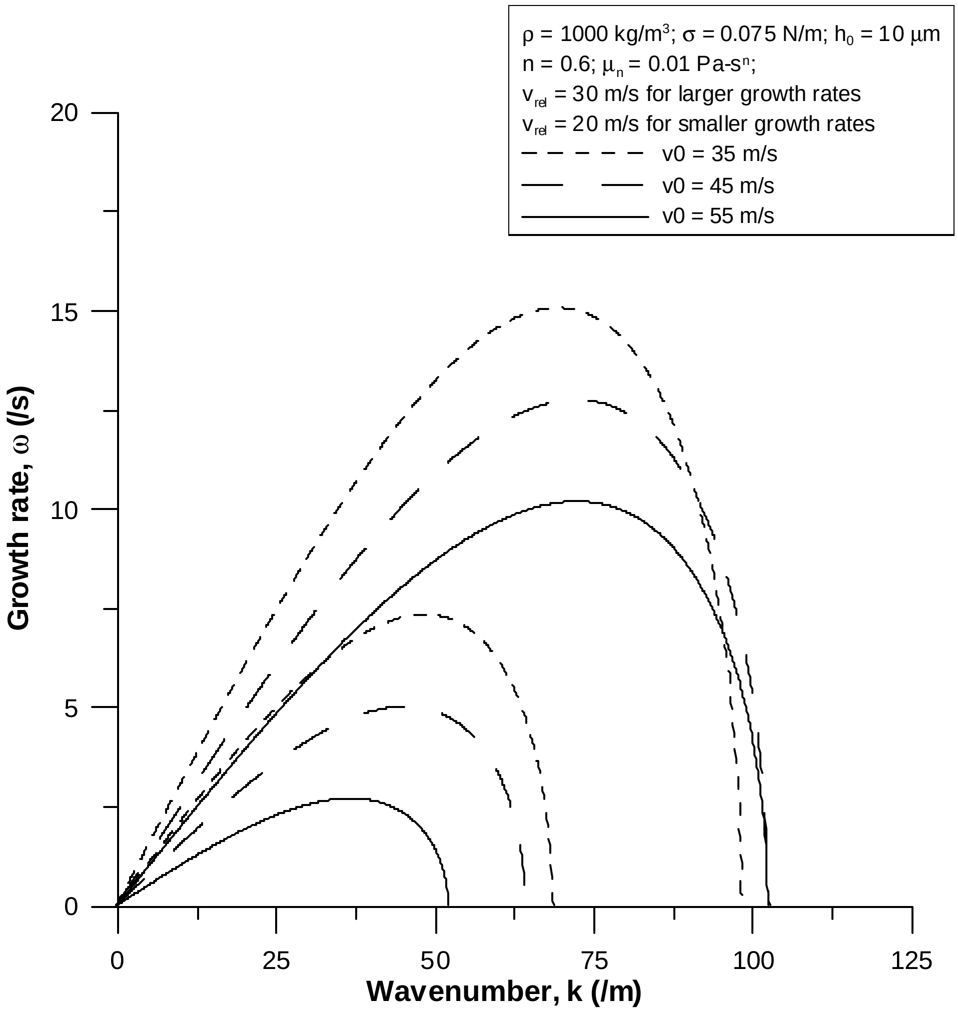

Figure 3. Growth rate (ω) vs. wavenumber (k) for constant relative velocity.

The behavior presented in Fig. 3 demonstrates that the relative velocity between the surrounding gas and liquid plays an important role in the spray formation process, but that it is not the only factor. For instance, when comparing the trio of 30 m/s relative velocity curves we see that the larger mean liquid velocity case yields the smallest peak growth rate of the three, produces slightly smaller drops (smaller wavenumber at maximum growth rate) than either of the other cases, but has the largest range of disturbance wavenumbers that can lead to breakup. In contrast, when comparing the 10 m/s relative velocity curves we see that the larger surrounding air velocity case has the largest peak growth rate of the three, gives rise to the smallest drops (largest wavenumber at maximum growth rate), and also has the largest range of disturbance wavelengths that can lead to breakup.

Fig. 4 has initial conditions identical to those of Fig. 1, with the exception that the flow behavior index is raised from 0.6 to 0.9. Upon a comparison of Figs. 1 and 4 data it is evident that an increase in n causes the maximum growth rate to decrease, the wavenumber at which the maximum growth rate occurs to decrease (drop size rises), and the range of disturbance wavenumbers that can lead to breakup to decrease as well. Thus, an increase in the power law index (n) causes the flow to become more stable. This conclusion is further reinforced by Fig. 5, where we see that a decrease in the power law index leads to a decrease in the effective viscosity thus causing the flow to become more unstable and leading to a smaller drop size.

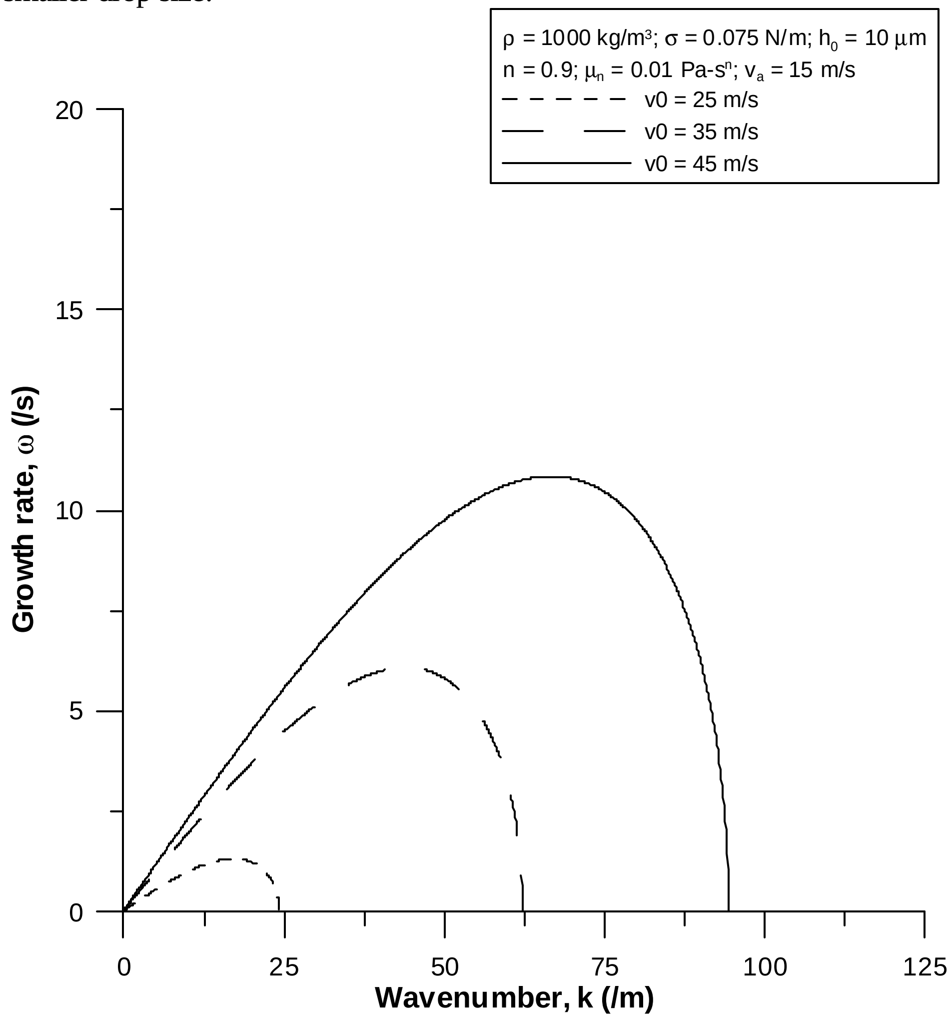

Figure 4. Growth rate (ω) versus wavenumber (k) for varying mean liquid velocities (v_0) and n = 0.9.

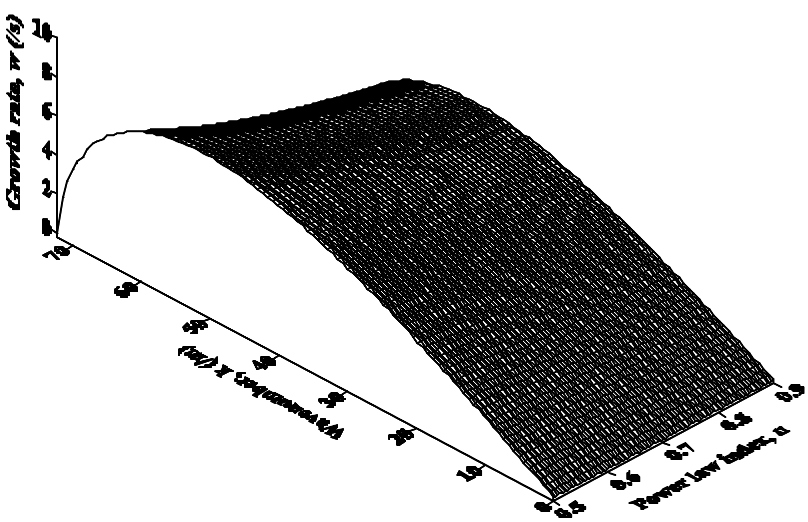

Figure 5. Growth rate (ω) versus wavenumber (k) and power law index n for h_0 = 10 μm, K = 0.01 Pa-s^n, ρ = 1000 kg/m^3, σ = 0.075 N/m, v_0 = 35 m/s and v_a = 15 m/s.

4. Summary and Conclusions

A linear stability analysis was performed on a cylindrical jet of a non-Newtonian power law fluid exiting into a moving gas. A steady state, symmetric velocity profile was assumed for the liquid and the effects of ambient gas velocity and mean liquid velocity on the breakup process were examined. We observe that the peak growth rate increases as the relative velocity increases, that the disturbance wavenumber at peak growth rate increases, and that increasing the relative velocity also increases the range of wavenumbers that can cause breakup. We also note that the relative velocity is not the only parameter determining the breakup process, but that the mean liquid and surrounding gas velocities are important as well. In particular, the wavenumber corresponding to the maximum growth rate decreases as v_a increases (at fixed v_0), and the range of wavenumbers causing breakup increases. Finally, when mean liquid velocity increases at fixed ambient gas velocity the growth rate increases, the wavenumber corresponding to the peak growth rate decreases, and the range of wavenumbers that can lead to breakup increases. We also observe that a decrease in the power law index makes the flow more unstable.

Our model does have limitations. It is not valid at ambient velocities above the mean liquid velocity. Further work will involve performing a non-linear analysis on the cylindrical jet.

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