# SPRAY FORMATION BY BI-COMPONENT LIQUID FLASHING: A THEORETICAL APPROACH

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Spray formation by bi-component liquid flashing through an injector has been studied by using a one-dimensional model. The injector consists of an inlet orifice, an expansion chamber, and a discharge orifice. In principle, in this method, a given mixture of two different types of liquids (a solvent and a propellant) flows through the inlet orifice into the expansion chamber and leaves it through the discharge orifice. The propellant, which has a higher vapor pressure than the solvent, undergoes a rapid boiling process (flashing process), which enhances a prompt disintegration of the solvent.
The proposed model considers the major relevant processes; the pressure drop at the inlet orifice, bubble growth inside the expansion chamber, and the pressure drop and mixture acceleration at the discharge orifice. The processes have been analyzed by using a one-dimensional model approach. Whilst the one-dimensional assumption cannot be fully justified, it enables straightforward analysis and yet provides realistic quantitative results.
It is postulated that the optimal chamber design should allow, at the same time, a high nucleation rate at the inlet orifice, and high relative velocity between the vapor phase and the liquid phase while the mixture is being discharged. It is concluded that minimum SMD is obtained when conditions lead toward choked flow.

## 1. Introduction

Spray formation of a bi-component liquid is widely used in household applications; odors, medical, and painting sprays, are few examples. This type of spraying method provides remarkably low SMD spray for a relatively low operating pressure (SMD<100μm for pressure difference of less than 200kPa [1]). As compared to spray formation by mechanical means, for the same operating pressure, spray formation by flash boiling is characterized by smaller mean diameter, higher homogeneity, wider cone angle and shorter penetration depth. These characteristics are needed for many applications; smaller and more homogeneous spray are important in almost any application, shorter penetration depth is important in fuel injection systems, where droplet wall impingement should be avoided, and lower injection pressure is important where safety is a major concern [2]. Spray formation by flash boiling provides the opportunity to generate the desired spray at low injection pressures.
A number of studies have been focused on finding the relationships between the injection properties and the final spray properties ([3] [1] [4]). In these works energy balance analyses were used to correlate the spray SMD (or $d_{50}$) to the initial conditions of the mixture for a presumably optimal injector design. A detailed review can be found in Witlox & Bowen [5].

Bar-Kohany and Sher [6], developed a one-dimension model for the flow inside the injector unit for the case of subsonic regime. They postulated that in a well-designed expansion chamber, a pre-specified void fraction has to be attained at the end of the expansion chamber. The latter is designed to yield this void fraction, subjected to the thermodynamic conditions of the entering mixture and orifices geometries.

The present analysis allows sonic conditions at the discharge orifice. In this case, a completely different flow regime takes place; the bubbles and the liquid bulk have different velocities, lower SMD can be achieved, and the chamber design has to be modified accordingly.

## 3. The model

### *3.1. Nucleation and bubble growth*

A given mixture of two different types of liquids, a solvent, $\alpha$, and a propellant, $\beta$, flows through an inlet orifice into an expansion chamber. The propellant, which has a higher vapor pressure, undergoes a nucleation process (Fig 1b):

(1) $\dot{n} = J_n \cdot \pi \cdot d_i \cdot l_i$ ; $J_n$ - nucleation flux

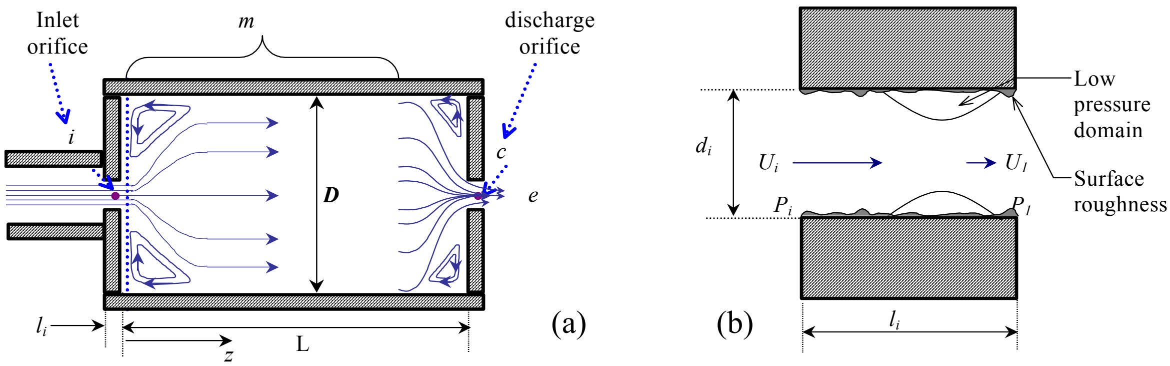


**Figure 1** (a) Injector design (b) Wall nucleation at the inlet orifice (based on Fig 1 in [8])

Since the value of the nucleation rate is very sensitive to the experimental conditions, and the bubble number density may vary over a number of orders of magnitudes, we have decided to consider their phenomenology significance rather than their absolute values (see details in [7]). Thus, since the nucleation rate depends exponentially on the superheat degree, we obtain:

(2) $J_n = c_1 \cdot exp\left(-\dfrac{c_2}{\Delta P^2}\right)$ ; $c_1, c_2$ - Constants

The superheat degree of the propellant:

(3) $\Delta P = y_\beta \cdot P_{sat,\beta}(T_m) - P_m$

Further we assume that the bubble growth is controlled by thermal diffusion, i.e. *Le<1*, then under quasi-steady conditions:

(4) $R_b(t) = R_b(t=0) + 2 \cdot \sqrt{\dfrac{3}{\pi}} \cdot Ja \cdot \sqrt{\alpha_\beta \cdot t}$ ; $R_b(\tau) >> R_b(t=0)$

Where the Jacob number reflects the superheat degree of the propellant:

(5) $Ja = \dfrac{T_m \cdot \rho_{l,\beta} \cdot C_{P,l,\beta} \cdot \Delta P}{h_{fg,\beta}^2 \cdot \rho_{v,\beta}^2}$

*3.2. Flow inside the expansion chamber and at the discharge orifice*

The velocity of the mixture inside the expansions chamber (m) is obtained through the continuity equation:

$$(6)\quad \frac{\rho_i \cdot U_i \cdot A_i}{U_{m,z} \cdot A_m} = \left[\beta \cdot \left(1 - x_\beta\right) \cdot \rho_{l,\beta} + \left(1 - \beta\right) \cdot \rho_\alpha\right] \cdot \left(1 - \varepsilon_{A,z}\right) + \beta \cdot x_\beta \cdot \rho_{v,\beta} \cdot \varepsilon_{A,z}$$

We further assume that the mass fraction of the liquid propellant, $\beta$, is fairly constant across the chamber. It follows that the vapor/liquid mass flow rate is negligible, thus:

$$(7)\quad U_{m,z} = U_i \cdot \frac{A_i}{A_m} \cdot \frac{1}{1 - \varepsilon_{A,z}}$$

$$(8)\quad U_i = \sqrt{2 \frac{P_i - P_m}{\rho_i}}$$

Next, we examine the flow towards the end of the chamber, and distinguish between two cases: (3.2.1) non-choked (subsonic) flow, and (3.2.2) choked flow.

*3.2.1 Non-choked flow at the discharge orifice*

For non-choked flow, the pressure at the discharge orifice equals the pressure in the atmosphere into which the spray is discharged. The flow towards the end of the chamber comprises a mixture of solvent, liquid propellant, and vapor propellant, that accelerates towards the discharge orifice. Since the ratio between the driving and inertia forces in the liquid is different from that of the vapor, their local velocities are essentially different, and slip between the two phases is expected. It is however noted that the mass flow ratio between the vapor and the liquid is around 1/50, and therefore the momentum that might be gained by the liquid due to drag forces between the two phases is merely negligible. Therefore, for non-choked flow:

$$(9)\quad U_{le} = \sqrt{2 \frac{\left(P_m - P_e\right)}{\rho_l}} = U_e$$

The pressure inside the expansion chamber, $P_m$, may be evaluated through the combined mass-energy conservation equation to yield:

$$(10)\quad C_{D,i} \cdot A_i \cdot \rho_i \cdot U_i = C_{D,e} \cdot A_e \cdot \rho_e \cdot U_e$$

$$(11)\quad \rho_e = \rho_{v,\beta} \cdot \varepsilon_e + \rho_l \cdot \left(1 - \varepsilon_e\right)$$

Further we assume $\rho_l \approx \rho_i$, and thus:

$$(12)\quad P_m = \frac{P_i + \left(\frac{C_{D,e} \cdot A_e \cdot \rho_e}{C_{D,i} \cdot A_i \cdot \rho_i}\right)^2 P_e}{1 + \left(\frac{C_{D,e} \cdot A_e \cdot \rho_e}{C_{D,i} \cdot A_i \cdot \rho_i}\right)^2}$$

*3.2.2 Choked flow at the discharge orifice*

The present model assumes separated-velocities frozen-flow, i.e. it considers slip between the two phases inside the discharge orifice, but no significant mass or heat transfer is allowed. The critical pressure ratio of the two-phase mixture resembles the critical ratio of the vapor

(Fig 2), and therefore it is assumed that the pressure profile along the streamlines is dominated by the vapor disposition [9].

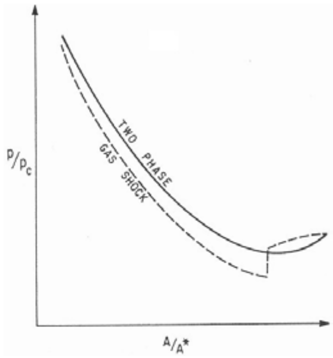


**Fig 2** Single and 2P overexpansion [9]

Thus, for choked flow, the pressure at the discharge orifice is determined, the liquid phase velocity may be estimated from this pressure, and since the densities ratio between the liquid and the vapor >>10, the liquid mass-flux determines the total mass-flux (Yang et al. [10]).
The velocity inside the expansion chamber is very low (for $A_i$ $/A$ $\approx 10^{-2}$, $\varepsilon_{\max} = \varepsilon_{A,L} \approx 0.5 \sim 0.8$, $U_{z=L}$ $\approx 10^{-1}$m/s) the velocity at the end of the chamber stays low comparing with the velocity at the discharge orifice. We further assume that the vapors follow the ideal gas law, and that they undergo a reversible expansion along a streamline, thus:

(13) $\dfrac{P_c}{P_m} = \left(\dfrac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}$ ; $P_c$ – Critical pressure inside the discharge orifice

(14) $\dfrac{\rho_{v,c}}{\rho_{v,m}} = \left(\dfrac{P_c}{P_m}\right)^{1/\gamma}$ ; $\rho_{v,L} = \rho_v(T_m) \xrightarrow{T_m = T_i} \rho_{v,i}$

(15) $a_v = \sqrt{\dfrac{2\gamma}{\gamma+1} \cdot R_\beta \cdot T_m}$ ; $a_v$ – sonic velocity of the vapor

Towards the end of the chamber the mixture converges into the discharge orifice. In the case of choked flow, where the vapors reach the appropriate sonic velocity, slip cannot be neglected, due to the fact that the pressure of the vapors equals to that of the liquid. The liquid's energy is estimated by Bernoulli's equation.

(16) $U_{l,e} = \sqrt{\dfrac{2\,P_m}{\rho_l}\left(1 - \dfrac{P_c}{P_m}\right) + U_{m,z}{}^2}$

Due to slip, the void fraction at the discharge orifice (*e*) is lower that at the end of the chamber (*L*) and is calculated by:

(17) $\rho_{v,L} \cdot U_{m,L} \cdot \varepsilon_{A,L} \cdot A_m = \rho_{v,c} \cdot a_v \cdot \varepsilon_{A,e} \cdot A_e$

(18) $\varepsilon_{A,e} = \dfrac{\rho_{v,i}}{\rho_{v,c}} \cdot \dfrac{A_i}{A_e} \cdot \sqrt{\dfrac{2(P_i - P_m)}{\rho_i}} \cdot \dfrac{\varepsilon_{A,L}}{1 - \varepsilon_{A,L}} \cdot \dfrac{1}{a_v}$ ; $\rho_i \approx \rho_l$

From continuity:

$$(19)\quad C_{D,i}\cdot A_i\cdot \rho_i\cdot U_i = C_{D,e}\cdot A_e\cdot\left[\rho_{v,c}\cdot\varepsilon_{A,e}\cdot a_v+\rho_l\cdot\left(1-\varepsilon_{A,e}\right)\cdot U_{l,e}\right]$$

Substituting equations (14) through (19) into equation (20) yields:

$$(20)\quad \begin{aligned}&\frac{C_{D,i}\cdot A_i}{C_{D,e}\cdot A_e}\cdot\sqrt{2\frac{P_i-P_m}{\rho_i}}=\frac{\rho_{v,i}}{\rho_i}\cdot\sqrt{\frac{2\left(P_i-P_m\right)}{\rho_i}}\cdot\frac{\varepsilon_{A,L}}{1-\varepsilon_{A,L}}\cdot\frac{A_i}{A_e}+\\&+\left(1-\frac{\rho_{v,i}}{\rho_{v,c}}\cdot\frac{A_i}{A_e}\cdot\sqrt{\frac{2\left(P_i-P_m\right)}{\rho_i}}\cdot\frac{\varepsilon_{A,L}}{1-\varepsilon_{A,L}}\cdot\frac{1}{a_v}\right)\cdot\sqrt{\frac{2\,P_m}{\rho_l}\left(1-\frac{P_c}{P_m}\right)+\frac{2\left(P_i-P_m\right)}{\rho_i}\cdot\left(\frac{A_i}{A_m}\cdot\frac{1}{1-\varepsilon_{A,L}}\right)^2}\end{aligned}$$

This equation is to be solved for $P_m$.

### *3.3. Optimal volume of the expansion chamber and optimal orifices' diameter ratio*

Knowing the properties of the mixture components inside the expansion chamber and in the discharge orifice, we can now estimate the optimal volume of the expansion chamber [6] and the optimal diameter ratio of the orifices.

From geometry consideration, the cross-sectional void fraction is:

$$(21)\quad \varepsilon_{A,z}=\frac{\dfrac{\dot n\cdot 2\cdot R_b}{U_{m,z}}\cdot\pi R_b^{\,2}}{A_m}=\frac{2\cdot\pi\cdot R_b^{\,3}\cdot\dot n}{U_{m,z}\cdot A_m}$$

Substituting $\varepsilon_{A,z}$ from Eq. (21) into Eq. (7) and solving for $U_{m,z}$ yields:

$$(22)\quad U_{m,z}=U_i\cdot\frac{A_i}{A_m}+\frac{2\cdot\pi\cdot R_b^{\,3}\cdot\dot n}{A_m}$$

The optimal volume is designed so that at the end of the chamber, the bubbles will touch one another, hence the continuous phase, the liquid, will become dispersed.

Substituting $R_b$ from Eq. (4) into Eq. (22), considering that $U_{m,z}=dz/dt$ and integrating from $z=0$ to $z=L$, and from $t=0$ to $t=\tau$, we obtain $L$, and along with $R_b(\tau)$ ($\varepsilon_{A,L}$ is known) we obtain an expression for the optimal volume of the expansion chamber:

$$(23)\quad V_{m,Optimal}=C_2\cdot\frac{\left(U_i\cdot A_i\right)^{5/3}}{J_a^2\cdot\alpha_\beta\cdot\dot n^{2/3}}\qquad ;\ C_2=C_1+\frac{32\cdot\pi}{5}\cdot\left(\frac{3}{\pi}\right)^{3/2}\cdot C_1^{\,5/2}$$

The residence time of the bubble inside the expansion chamber, $\tau$, may be derived from Eqs. (4) and (23), to yield:

$$(24)\quad \tau=C_1\cdot\frac{1}{Ja^2}\cdot\frac{1}{\alpha_\beta}\cdot\left[\frac{U_i\cdot A_i}{\dot n}\right]^{\frac{2}{3}}\quad ;\ C_1=\frac{\pi}{12}\cdot\left(\frac{1}{2\cdot\pi}\right)^{\frac{2}{3}}\cdot\left(\frac{\varepsilon_{A,L}}{1-\varepsilon_{A,L}}\right)^{\frac{2}{3}}$$

Since shear stresses between the liquid and the surrounding enhance the breakup process, it is expected that higher shear stresses would yield smaller mean drop size. Maximum shear stresses occur when the velocity at the discharge orifice is maximal, e.g., the pressure drop across the discharge orifice is high enough to result in a close-to-choked flow. This is achievable when the $d_i/d_e$ ratio is high. The minimum ratio of the orifices' diameters for choked flow could be estimated by:

$$(25)\quad \frac{P_e}{P_m}\le\frac{P_c}{P_m}\quad\xrightarrow{P_m,\ eq.(13)}\quad\left(\frac{d_i}{d_e}\right)_{choking}\ge\left[\frac{\rho_e}{\rho_i}\cdot\sqrt{\frac{1-\dfrac{P_c}{P_m}}{\dfrac{P_c}{P_m}\cdot\dfrac{P_i}{P_e}-1}}\right]^{1/2}$$

## 6. Conclusions

A one-dimensional model approach is proposed. It considers the major relevant processes; the pressure drop at the inlet orifice, bubble growth inside the expansion chamber, and the pressure drop and mixture acceleration at the discharge orifice. The optimal chamber geometry that results in minimum SMD has been defined.
It is postulated that the optimal chamber design should allow:

1. High nucleation rate at the inlet orifice (maximum pressure drop at the inlet orifice),
2. High relative velocity between the vapor phase and the liquid phase while the mixture is being discharged (pressure drop at the discharge orifice which leads toward choked flow),
3. Bubbles growing till one touches another at the discharge orifice (defines the chamber volume in terms of the superheat degree),
4. Very well mixing inside the expansion chamber (affected by the L/D chamber ratio).

## 7. References


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