Numerical analysis of one-dimensional atomization model described by Eulerian type equations

Kenji Amagai, Norihiko Ueno and Masataka Arai

Dept. of Mechanical System Engineering, School of Engineering, Gunma University

1-5-1 Tenjin-cho, Kiryu 376-8515, Japan

Abstract

In our previous reports, an Eulerian type two-fluid model for gas-liquid two-phase flow in concern with the atomization process was derived. General theories of two-phase flow dynamics were used for the derivation of it. Derived equations had source term related to the increase of surface area. In this report, one-fluid model was derived from the two-fluid model. Simple breakup model, which was based on the energy balance during the droplet breakup process, was used in the numerical analysis. Atomization efficiency was introduced to describe the breakup phenomena and the breakup process was numerically simulated using this atomization efficiency. Interfacial area concentration could be obtained as a function of flow direction. And the effect of physical properties on atomization characteristics was discussed.

1. Introduction

An atomization phenomenon was a physical process changing from the continuous liquid flow with free surface to the dispersed gas-droplet two-phase flow. Liquid flow before the atomization was described by the Eulerian type conservation equations, and some numerical methods for solving the equations had been presented. For the two-phase flow, some mathematical models had been developed to analyze the flow characteristics [1-3]. A lot of numerical studies had been carried out based on the two-phase flow models. Obtained results agreed with experimental results. However, mathematical modeling of the atomization process had not yet established sufficiently because of the complexity of atomization phenomena.

The modeling method of the two-phase flow was classified into two different types, namely a Lagrangian-Eulerian method [1] and an Eulerian-Eulerian method [4-6]. Usually, the spray flow was calculated by the Lagrangian-Eulerian method. In this method, the motions of many droplets or droplet groups were analyzed by a Lagrangian type equation and the flow of surrounding gas was described by the Eulerian type equation by considering an interaction between gas and droplets. However, it is very difficult to obtain the initial conditions of the calculation because the initial state of the spray is related to the complex atomization phenomena.

If the atomization process can be introduced into the Eulerian type equation, the whole

flow fields including the pre-atomization flow, atomization area and the spray flow after the atomization are simply described by the Eulerian type equations. This mathematical treatment is very effective to develop the numerical scheme because the same algorithms can be applied to the calculations for both of liquid and gas phases.

In the previous report, we derived a two-fluid model of the droplet and gas flows with atomization process [7]. Moreover, we derived a one-fluid model for two-phase mixture consisted of droplet and gas phases [8]. And a break-up model based on the energy balance of surface area was introduced in the one-fluid model [9]. In this paper, atomization efficiency was newly introduced and a one-dimensional atomization problem was discussed. Namely, atomization phenomena from a few large droplets to many small droplets in a spray flow were analyzed by using the one-fluid model. Effects of atomization efficiency and physical properties on atomization behavior were investigated numerically.

2. Basic equations of Eulerian model

The basic concept of Eulerian-Eulerian model is expressed in Fig.1. Two conservation equations for droplet phase and gaseous phase were used to describe the two-phase flow [7]. This model is called as two-fluid model. By coupling of two conservation equations in the two-fluid model, governing equations of a one-fluid model were derived [8]. A mixture flow of droplets and gas phases was considered as the two-phase flow that was simply characterized by physical properties of the mixture.

In the one-fluid model, the density and velocity of the mixture are defined as $\rho = \alpha^l \rho^l + \alpha^g \rho^g$ and $\rho \mathbf{v} = \alpha^l \rho^l \mathbf{v}^l + \alpha^g \rho^g \mathbf{v}^g$. Where, superscript indicates a phase (l: liquid phase, g: gaseous phase), α is a volume fraction, ρ is a density and $\mathbf{v}$ is a velocity.

Mass conservation equation is described as follows.

$$\partial t \rho + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{1}$$

This equation coincides with the equation of continuity for a single phase. Source term does not appear in the equation because there is no mass generation during the atomization processes.

Momentum conservation equation is described as follows.

$$\partial t(\rho \mathbf{v}) + \nabla \cdot \left(\rho \mathbf{v}\mathbf{v} + \frac{\alpha^l \alpha^g \rho^l \rho^g}{\rho} \mathbf{u}\mathbf{u} - \mathbf{T} \right) = \beta \langle \sigma \kappa \ \mathbf{n} \rangle + \sum_p \left(\beta^c{}_p h_p \langle \sigma \kappa \ \mathbf{n} \rangle \right) \tag{2}$$

The second term in right hand side is a source caused by the atomization. $\mathbf{u}$ is a relative velocity between droplet and gas phases, $\mathbf{T}$ is a stress tensor, β is a surface area concentration, σ is a surface tension, κ is a radius of curvature, $\mathbf{n}$ is a normal unit vector on the droplet surface, $\beta^c{}_p$ is an increase of surface area concentration by the atomization, h_p is a dimensionless time of atomization process, the symbol $\langle \ \rangle$ is a spatial average and p is integers which indicates the droplet group related to the disintegration. In the case of no atomization, this term becomes zero.

By using Eqs.(1) and (2), equations of one-dimensional steady flow were derived. The mass and momentum conservation equations can be considered in one-dimensional steady flow. These equations are described as:

$$\frac{d}{dx}(\rho v_x) = 0 \tag{3}$$

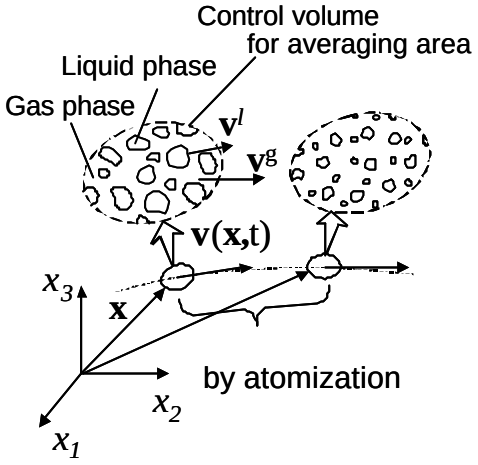

Fig.1 Eulerian model of two-phase mixture

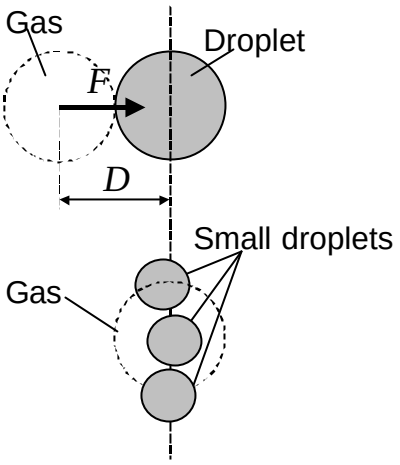

Fig.2 Derivation of surface tension force

$$\frac{d}{dx}\left(rv_x^2 + \frac{r^l r^g a^l a^g}{r} u_x^2 + P - 2m\frac{dv_x}{dx} \right) = b\langle sk\ \mathbf{n}\rangle\Big|_x + \sum_p \left(b_p^c h_p \langle sk\ \mathbf{n}\rangle \right)\Big|_x \qquad (4)$$

Where P is a pressure, m is a viscosity of gas and a subscript $|_x$ indicates an x-component. Eq.(3) is easily integrated and following equation is obtained.

$$rv_x = const. \equiv j_m \qquad (5)$$

If a droplet is spherical, the first term of the right hand side in Eq.(4) varnished from the force balance of surface tension. The second term of the right hand side cannot neglect when the atomization occurs in the flow fields. $\langle sk\ \mathbf{n}\rangle$ indicates the spatially averaged force of the surface tension in the control volume. Unbalance of surface tension force applied on the control volume appears in the x-direction by the atomization as shown in Fig.2. $Sb_p^c h_p$ relates to the atomization rate g and is rewritten as $(1/2)ng\Delta x$. From this model, the right hand side in Eq.(4) becomes the following form.

$$\sum_p \left(b_p^c h_p \langle sk\ \mathbf{n}\rangle \right)\Big|_x \cong \frac{1}{3}\frac{sn^2 g^2}{v_x^{l\,2}}\Delta x\,. \qquad (6)$$

Where, n is a number density of the droplet, g is an atomization rate, and $v^l{}_x$ is an x-direction velocity of droplet.

Since the mixture flow should be satisfied the mass-conservation equations of droplet and gas phases, following relations are also required.

$$a^l r^l v_x^l = const. \qquad (7)$$

$$a^g r^g v_x^g = const. \qquad (8)$$

In order to determine the dynamic behavior of the liquid phase, the equation of the droplet motion is used.

$$v^l \frac{dv^l}{dx} = \frac{3}{4}\frac{r^g}{r^l}\frac{u_x|u_x|C_D}{D} \qquad (9)$$

Here we assume that the liquid phase is consisted of the spherical droplets. D is a droplet diameter and C_D is a drag coefficient. The drag coefficient is obtained as the function of droplet Reynolds number Re_p.

$$C_D = 240 (Re_p \le 0.1031),$$

$$= \frac{24}{Re}(1 + 0.15 Re_p^{0.687})(0.1031 < Re_p < 989), \quad (10)$$

$$= 0.44 (Re_p \ge 989)$$

Atomization model used here is considered as follows. As shown in Fig.3, we assume that only a spherical mass of gas (gas ball), which has a same volume of the droplet, influences on a droplet. A characteristic length related to the interaction between gaseous ball and droplet is approximated by *D*. Therefore, the energy related to this interaction is simply described as *FD*. Here *F* is a drag force. And we assume that the surface energy increase is proportional to the *FD*. From these assumptions, finally we obtain the following relation.

$$g = B \frac{2F\nu^l}{s} \quad (11)$$

Here, we introduce an atomization efficiency *B*. This value indicates a ratio between input energy by the drag force applied on the droplet and surface energy. Here we use a drag force of spherical droplet.

$$F = \frac{p}{8} r^g u_x |u_x| D^2 C_D \quad (12)$$

The termination of the atomization is simply decided by the critical Weber number of single droplet [10].

$$We_{crit} = \left(\frac{r^l u^2 D}{s} \right)_{crit} = \frac{8}{C_D} \quad (13)$$

In order to solve Eq.(4) numerically, an increment of v_x for every Δx step was derived. The fundamental variable in the governing equations of one-fluid and one-dimensional model was the mixture velocity v_x. In this calculation, mixture velocity v_{xi} at $x=x_i$ was obtained from the variables at $x = x_{i-1}$. Derived v_{xi} was rewritten with considering the Eqs.(5), (7) and (8) at every step. Numerical conditions of this study are shown in Table 1. Atomization phenomena of low-speed droplets injected into the high-speed gas flow were discussed as shown in Fig.4.

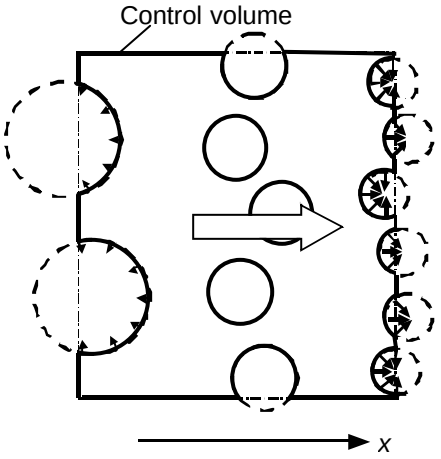

Fig.3 Droplet breakup model

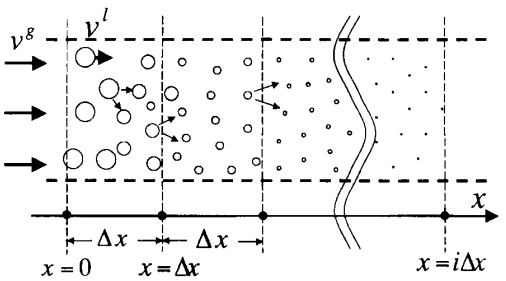

Fig.4 One-dimensional atomization problem

Table 1 Numerical conditions

Initial droplet diameter, D_0	0.005m
Droplet velocity at x=0, v^l_0	1.0m/s
Gaseous velocity at x=0, v^g_0	100m/s
Volume fraction of droplet at x=0, α^l_0	0.01, 0.2
Atomization efficiency, B	0 (without atomization) 0.005 – 0.02 (with atomization)

3. Results and discussion

3.1. Atomization characteristics of droplets in a high-speed gas flow

Gas and droplet velocities were derived as functions of x. Obtained results were summarized in Fig.5. The atomization efficiency B was given at 0.01. For the comparison, non-atomization case (B=0) is plotted in the figure. v^g and v^l approach to v_∞ for both cases of atomization and non-atomization. Where v_∞ was derived theoretically. From the figure, it was confirmed that the droplet velocity of the atomization case (B=0.01) v^l quickly approached to v_∞ because the droplet diameter became smaller with the atomization process and was easily accelerated by the gaseous flow.

Effects of atomization efficiency B on interfacial area concentration b [m^2/m^3], droplet diameter D and increase of interfacial area b_b [m^2/m^3] are summarized in Fig.6. Interfacial area concentration b means a total surface area (m^2) per unit volume (m^3). An increase of interfacial area b_b indicates the total increasing area (m^2) per unit volume (m^3). As shown in this figure, all the variables can be described as the function of spatial coordinate (in this case, the coordinate is x-direction). This is one of advantage in Eulerian method. Maximum interfacial area concentration b_{max} was defined as shown in the figure.

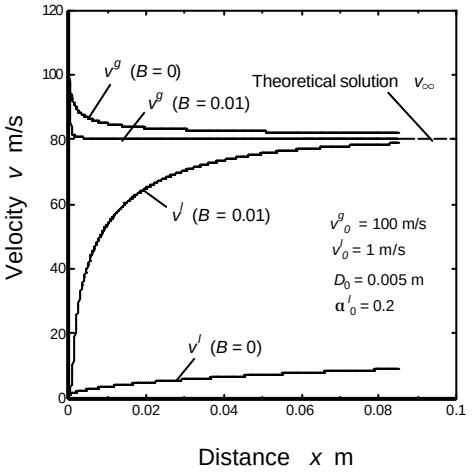

Fig.5 Velocity change of droplet and gaseous flow

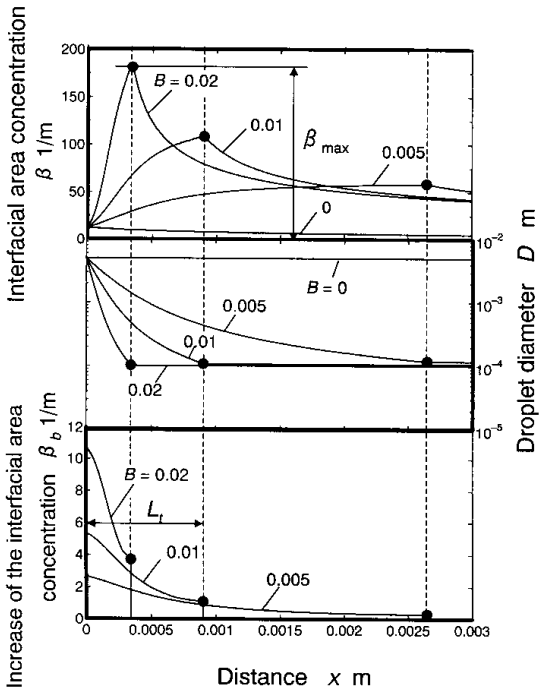

Fig.6 Effects of B on b, D and b_b

Termination points of the atomization are plotted by black circles (•). Termination length L_t was defined as shown in the figure. The physical meaning of L_t is the position where the atomization process is terminated. They were strongly affected by B. But the final droplet diameter was not affected by B. Interfacial area concentration b had a maximum point. In the case of $x<L_t$, the surface area became large by the atomization. In $x<L_t$, the atomization was terminated and the droplets were accelerated to the flow direction. Therefore the surface area concentration b became low. b_b was a maximum at x=0 and was decreased with an increasing of x. After the termination point, b_b became zero.

Figures 7 and 8 show the effect of B on the atomization characteristics. b_{max} becomes large with increasing of B. This result is reasonable because the atomization is enhanced with increasing of B. Termination length L_t decreases with increasing of B.

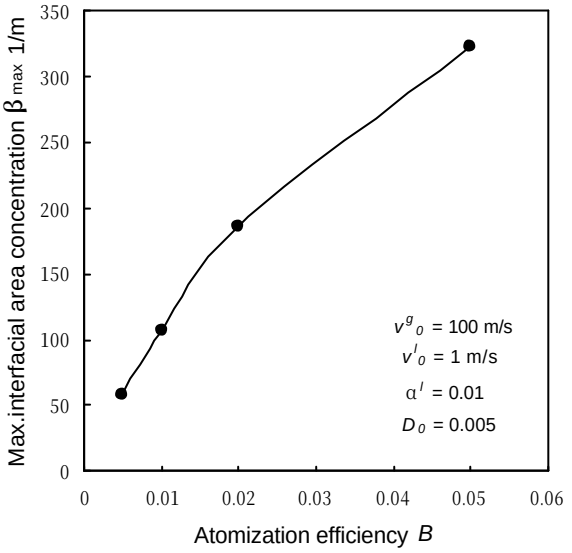

Fig.7 Effect of B on b_{max}

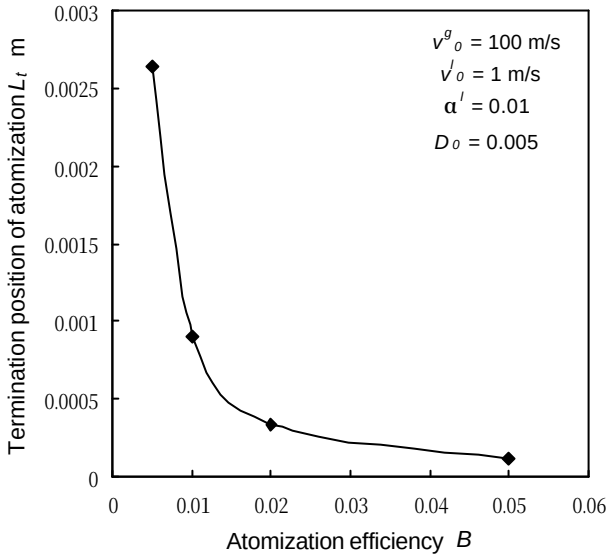

Fig.8 Effect of B on L_t

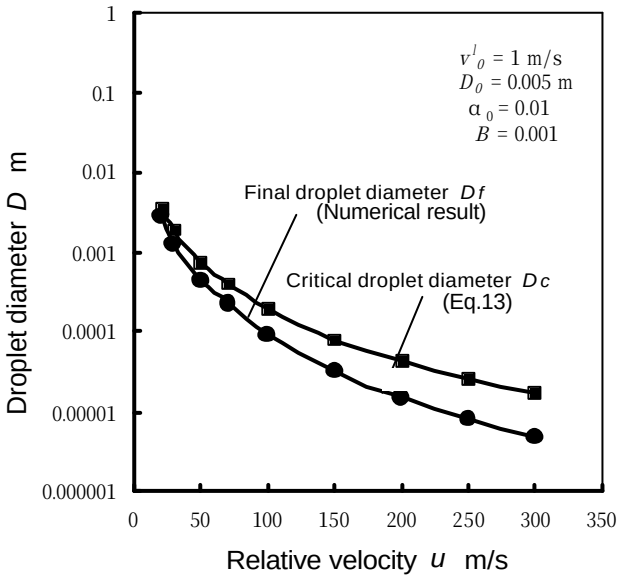

Fig.9 Influence of relative velocity u on final droplet diameter D_f

3.2. Droplet diameter

Final droplet diameter D_f was derived under various conditions. Figure 9 shows the effect of relative velocity u on D_f. The critical droplet diameter D_c, which was calculated from Eq.(13), was plotted in the figure. Numerically derived final droplet diameter D_f was larger than D_c. In Eq.(13), relative velocity u was fixed at v^g_0-v^l_0. On the other hand, the effect of droplet acceleration was included in our model. Therefore, the D_f became larger than D_c.

3.3.Effect of physical properties on atomization characteristics

Effects of gas density r^g and liquid density r^l on the maximum interfacial area concentration b_{max} were investigated. Results were shown in Figs.10 and 11. The r^l was strongly depended on the b_{max} rather than the r^g. Increase of liquid density causes the decrease of droplet acceleration. Therefore, the interfacial area per unit volume becomes large with an increase of r^l.

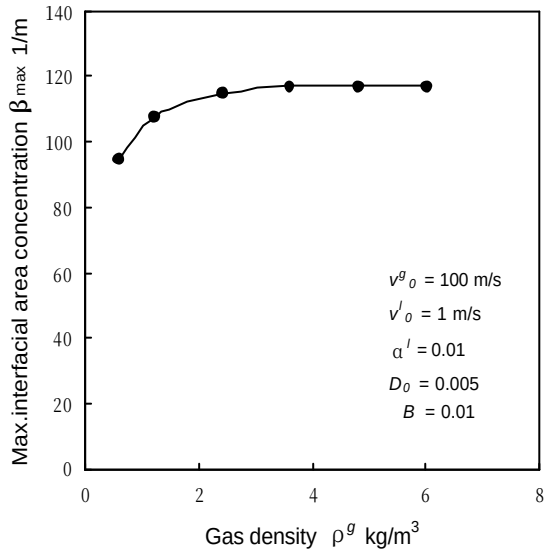

Fig.10 Effect of r^g on b_{max}

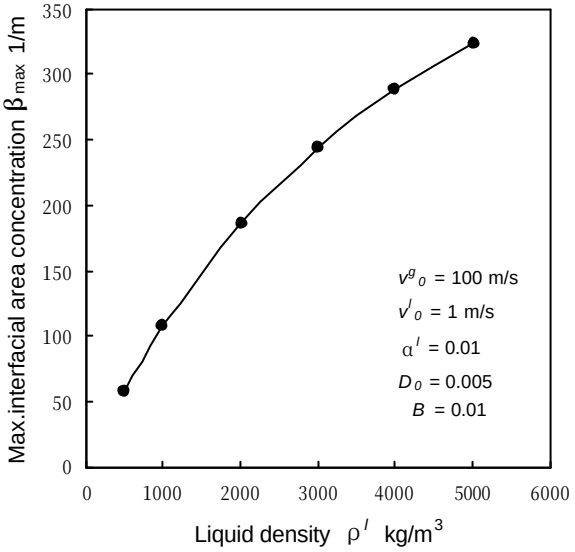

Fig.11 Effect of r^l on b_{max}

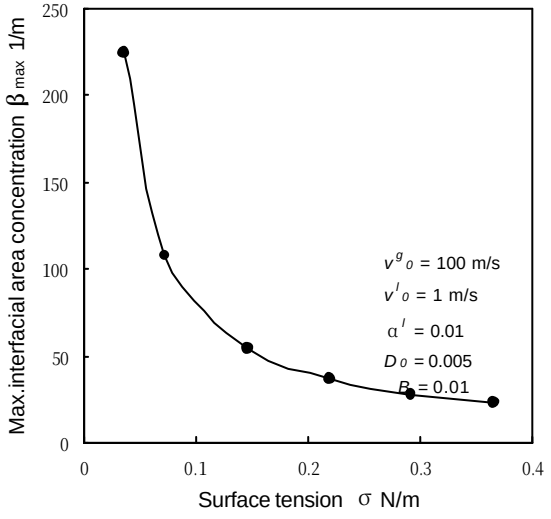

Fig.12 Effects of s on b_{max}

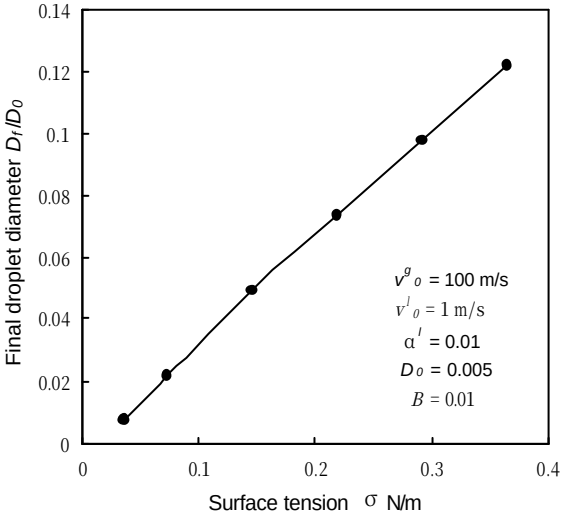

Fig.13 Effects of s on D_f

Effects of surface tension s on the b_{max} and final droplet diameter D_f were investigated. Results were shown in Figs.12 and 13. b_{max} decreased and D_f increased with increasing the surface tension. These results were reasonable because an increase of surface tension leads to the stabilization of the droplet.

4. Conclusions

Atomization efficiency was introduced to describe an atomization phenomenon. Eulerian type conservation equation was used to analyze the atomization phenomena. Even though the one-dimensional model was simple, the reasonable results were obtained for understanding of atomization process. Especially, the spatial variation of some variable such as droplet velocity, interfacial area concentration and droplet diameter could be easily described.

References

[1] C.T.Crowe, M.P.Sharma, D.E.Stock, The Particle-Source-In Cell (PSI-CELL) Model for Gas-Droplet Flows, Journal of Fluids Engineering, Transactions of the ASME, pp.325-332, (1977).

[2] C.F.Edwards, The Point-Particle/Continuum-Field Theory of Spray Flows, ICLASS-'97, pp.673-681, (1997).

[3] M.R.Maxey, B.K.Patel, E.j.Chang, L.-P.Wang, Simulations of dispersed turbulent multiphase flow, Fluid Dynamics Research, 20, pp.143-156, (1997).

[4] D.A.Drew, Mathematical Modeling of Two-phase Flow, Ann. Rev. Fluid Mech, 15, pp.261-291, (1983).

[5] I.Kataoka, Local Instant Formulation of Two-phase Flow, Int. J. Multiphase Flow, Vol.12, No.5, pp.745-758, (1986).

[6] D.A.Drew and L.A.Segel, Averaged Equations for Two-Phase Flow, Studies in Applied Mathematics, Vol.L, No.3, (1971).

[7] K.Amagai and M.Arai, Derivation of Eulerian Type Conservation Equations for Atomization Process, Proc. of ICLASS, pp.839-846, (2000).

[8] N.Ueno, K.Amagai and M.Arai, Theoretical Study of One-dimensional Atomization Problem, Proc. of ILASS-Asia, pp.140-145, (2001).

[9] N.Ueno, K.Amagai and M.Arai, Theoretical Study of One-dimensional Atomization Problem (2nd Report:Effect of Atomization Efficiency on Breakup Processes), Proc. of ILASS-Asia, pp.63-69, (2002).

[10] A.H.Lefebre, Atomization and Sprays, Hemisphere Publishing Corporation, pp.29-34, (1989).