Analysis of Double-Peak Spray Drop-Size Distribution from the Application of the Maximum Entropy Formalism

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A recent application of the Maximum Entropy Formalism (MEF) on liquid atomization problems led to the development of a mathematical volume-based drop-size distribution. The application of this function to sprays produced by ultrasonic atomizers revealed that the three parameters introduced by the mathematical distribution were in direct connection with the parameters controlling the physics of the involved atomization process. In the present paper, a similar analysis is reported for drop-size distributions of sprays produced by cavity nozzles. The particularity of these distributions is that they exhibit two maxima. Their representations were achieved thanks to a combination of two mathematical functions. The results show first that the mathematical function deduced from the application of the MEF is very well adapted to represent the bimodal drop-size distribution. Second, the values of the 5 parameters introduced by the mathematical distribution provide some insight into the basic mechanism involved in the atomization process.

1. Introduction

The drops emitted from an atomization process have different sizes. These sizes are mathematically represented by probability distribution or by probability density function. Among other parameters, this function depends on the disintegration process that takes place on the issuing liquid flow. At low injection pressure, this dependence involves the shape of the issuing liquid flow. Drop-size distributions of liquid sprays are often mono-modal, i.e., they exhibit a single maximum. This is the case for instance for sprays produced by cylindrical liquid jets, flat liquid sheets (if the edges are not taken into account) or a conical liquid sheet produced by swirl atomizer. However, this might not be the case anymore when the shape of the liquid flow is more complex as it can be observed for sprays produced by cavity nozzle used at low injection pressure [1].
The purpose of this work is to test the ability of the mathematical drop-size distribution deduced from a previous application of the Maximum Entropy Formalism (MEF) to represent bimodal distributions. Furthermore, the physical relevance of the use of this mathematical function will be discussed through the analysis of the parameters it introduces.

2. Experimental work

The analysis presented in this paper is carried out on a series of low-pressure cavity-nozzle injectors. Cavity nozzles are composed of a superposition of three disks as schematized in Fig. 1. Each disk is perforated by a circular hole. The hole of the cavity disk (disk 2) is much

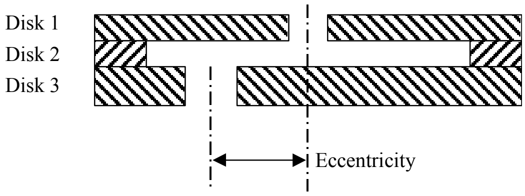

Fig. 1 Design of a cavity nozzle

larger than the two other disk holes. The concept of this nozzle rests on the production of a complex issuing liquid flow with energetic structures. A previous investigation [1] reported that the atomization efficiency of a cavity nozzle shows great variations when the eccentricity of the nozzle varies. According to some studies found in the literature, the improvement of primary atomization is due to the production of a turbulent issuing flow [2, 3] thanks to the complex trajectories imposed by the nozzle eccentricity. However, other investigation concluded to a negligible influence of the turbulence on the atomization process itself [4]. For the nozzle considered in the present investigation, we observed that the influence of the nozzle

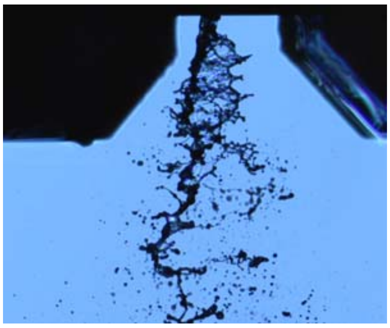

Fig. 2 Liquid system issuing from a cavity nozzle with a non-zero eccentricity.

	Disk 1	Disk 2	Disk 3
Thickness (µm)	177	75	76
Hole diameter (µm)	300	2,254	180
Available eccentricities E_c (µm)			200; 500; 700; 900

Table 1 Dimensions of the nozzle disks

E_c	**ΔP_i = 3-bar**		**ΔP_i = 4-bar**		**ΔP_i = 5-bar**	
(µm)	**V_q (m/s)**	**We**	**V_q (m/s)**	**We**	**V_q (m/s)**	**We**
200	19.3	3.4	22.1	4.4	24.7	5.5
500	18.9	3.2	21.6	4.2	24.0	5.2
700	18.4	3.0	21.1	4.0	23.2	4.8
900	18.5	3.1	21.3	4.1	23.8	5.1

Table 2 Mean velocity and Weber number of the issuing liquid flow

eccentricity was mainly to modify the shape of the issuing liquid flow [1]. As shown in Fig. 2, although the nozzle discharge orifice is a circular hole, the issuing liquid flow shows an important spatial expansion and adopts the behavior of a liquid sheet bordered with a liquid rim.

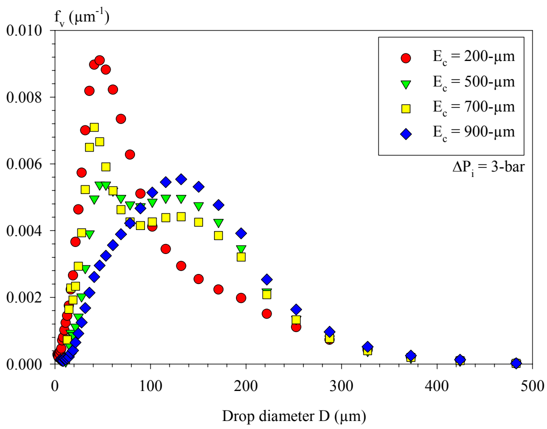

Fig. 3 Volume-based drop-size distributions measured at 3-bar

The series of injectors used in the present investigation differ by their eccentricity. Their dimensions are given in Table 1. All the experiments were conducted with a single fluid with the following physical properties: density 760-kg/m^3, surface tension 24-mN/m, dynamic viscosity 9.4.10^{-4}-kg/(ms). The injection pressure was varied from 3-bar to 5-bar and the issuing mean velocity V_q and Weber number We (calculated on the basis of the surrounding air density and of the diameter of the discharge orifice) were measured and calculated for each

case. The results are given in Table 2. For all cases, we can notice the small value of the Weber number indicating a weak influence of the aerodynamic forces on the disintegration process.
The drop-size distributions were measured with a Malvern Spraytec equipped with a 200-mm focal length collecting lens. The measurements were performed at 50-mm from the injector where it was checked that the 10-mm diameter laser beam embraced all the spray. The distributions obtained at 3-bar are presented in Fig. 3. It can be seen in this figure that the eccentricity of the nozzle deeply influences the resulting drop size distribution. Furthermore, it can be noted that the distributions are bimodal. This behavior is believed to be relevant since the ability of the Malvern Spraytec to measure bimodal drop-size distribution has been carried out in a study conducted in parallel [5]. Whatever the eccentricity, the two peaks of the distribution are more or less the same, namely, 50-µm and 150-µm. Bimodal spray drop-size distributions are often related to the presence of two disintegration processes. This coincides with what is observed in the present study (Fig. 2).
The purpose of the present study is to analyze the measured distributions through the application of the Maximum Entropy Formalism (M.E.F.) with three objectives. First, to examine the ability of the mathematical distribution deduced from the application of the M.E.F. to represent the measured drop-size distributions. Second, to develop a model to predict drop-size distributions on the basis of the M.E.F. Third, to investigate the possibility of extracting physical information related to the disintegration process from the M.E.F. analysis.

3. Application of the Maximum Entropy Formalism

The application of M.E.F. on liquid atomization problem led to the development of the following mathematical distribution to represent volume-based spray drop-size distribution [6]:

$$f_v(D) = q\frac{\Gamma^4\left(\frac{5}{q}\right)}{\Gamma^5\left(\frac{4}{q}\right)}\frac{\left(D-D_0\right)^3}{\left(D_{43}-D_0\right)^4}\exp\left[-\left(\frac{\Gamma\left(\frac{5}{q}\right)}{\Gamma\left(\frac{4}{q}\right)}\left(\frac{D-D_0}{D_{43}-D_0}\right)\right)^q\right] \tag{1}$$

where D is the drop diameter, q, D_0 and D_{43} are three parameters, and Γ stands for the classical Gamma function. The function f_v given by Eq. (1) is a reduction of the four-parameter Generalized Gamma function. The diameter D_{43} is the arithmetic mean of the volume-based drop size distribution. The parameter D_0 represents the diameter of the smallest drop of the spray. In many situations, this diameter can be assumed to be equal to zero, reducing the number of parameter to two. The function given by Eq. (1) can fit mono-modal drop-size distribution only. However, volume-based drop-size distributions with two peaks can be represented by a combination of single-peak drop-size distributions, namely:

$$f_v(D) = \alpha f_{v1}(D) + (1-\alpha)f_{v2}(D) \tag{2}$$

where f_{v1} and f_{v2} are either two or three-parameter Generalized Gamma function and α is a mixing parameter ($0 \leq \alpha \leq 1$). Thus, the mathematical distribution f_v given by Eq. (2)

depends on five, six or seven parameters according to the number of parameters introduced by f_{v1} and f_{v2}.
The application of the function f_v given by Eq. (2) consists in determining the parameters it contains. From a mathematical point of view, this could be achieved by using any combination of independent characteristics of the distribution such as mean drop diameters for instance. However, the resulting mathematical function is highly dependent on the used characteristics and the desired solution is usually obtained for a unique set of information [7]. This problem is overcome here by determining the parameters on the basis of a numerical procedure based on the use of the Kullback-Leibler number I defined by:

$$I = \sum_i p_{ei} \ln\left(\frac{p_{ei}}{p_i}\right) \tag{3}$$

where p_i and p_{ei} are two probability distributions. The number I constitutes a measure of the nearness of the two probability distributions [7]. In the present application p_{ei} represents the experimental probability distribution and p_i stands for the mathematical probability distribution. They are calculated by:

$$\begin{cases} p_{ei} = m_v(D_i)\Delta D_i \\ p_i = \int_{\text{Class } i} f_v(D)dD \end{cases} \tag{4}$$

where $m_v(D_i)$ and ΔD_i represent the measured volume fraction in class i and the width of this very class, respectively, and $f_v(D)$ is given by Eq. (2). The value of I (Eq. (3)) is a function of the parameters introduced by $f_v(D)$ and it is minimum when the probability distribution p_i is as near as possible to the experimental probability distribution p_{ei}. Therefore, for each situation, a numerical procedure seeks the best set of parameters that minimizes the number I. This calculation is conducted with the Matlab software.

4. Analysis

Previous applications of Eq. (1) on measured drop-size distribution [8-10] reported that the parameter q was constant provided that the atomization process is constant. The first step of the present analysis consists in seeing whether the distributions examined here can be represented by constant values of q_1 and q_2, the two parameters introduced by f_{v1} and f_{v2} respectively. To achieve this, the parameters of the mathematical function given by Eq. (2) are determined for each measurement according to the procedure described in the previous section. As far as the number of parameters to be taken into account is concerned, two cases are considered. For the peak of the small drop population, it is assumed that $D_0 = 0$. Thus, the function f_{v1} dedicated to this population, introduces the parameters q_1 and $D_{43,1}$. For the big drop population, the function f_{v2} is tested with two (q_2, $D_{43,2}$) and three (q_2, D_0, $D_{43,2}$) parameters. Mathematical distributions based on 5 or 6 parameters are tested for each case.
Figure 4 presents comparisons between mathematical and measured drop-size distributions. It can be first observed in Fig. 4 that the mathematical distribution (Eq. (2)) obtained by combining two single peak distributions (Eq. (1)) appears very much adapted to represent the measured drop-size distributions. Second, for all the investigated situations, it is found that the 5 and 6-parameter MEF distributions provide an equivalent fit. They are graphically

indissociable. Thus, in the following, it is assumed that the 5 parameters q_1, $D_{43,1}$, q_2, $D_{43,2}$ and α are sufficient to represent the drop-size distributions.

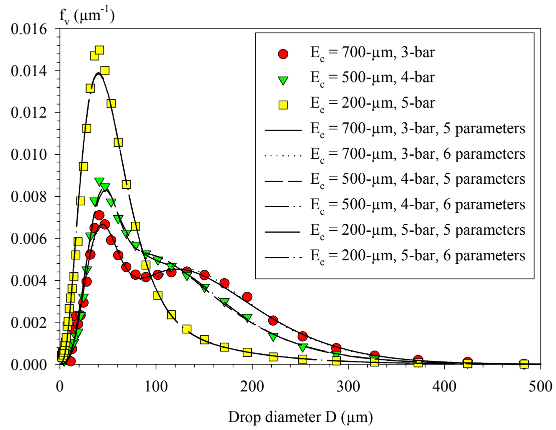

Fig. 4 Examples of comparison between mathematical and measured distributions (5 and 6-parameter applications)

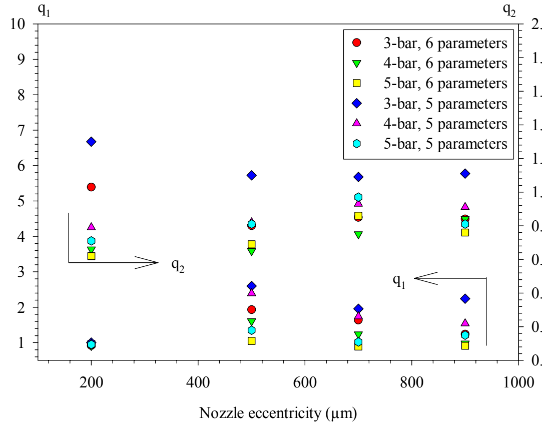

Fig. 5 Values of the parameters q_1 and q_2 of the 5 and 6-parameter MEF distributions

Figure 5 presents the values of parameters q_1 and q_2 resulting from the application of the 5 and 6-parameter MEF distributions. It shows that whatever the injection pressure and the number of parameters introduced, both q_1 and q_2 are almost independent of the nozzle eccentricity. As far as the injection pressure and the number of parameters are concerned, their influence shows no clear trend. One can notice however that for the nozzle eccentricity of 200-µm, the parameter q_1 was found constant whatever the injection pressure and the number of parameters used. Considering the fact that the results presented in Figs. 4 and 5 are free of any physical considerations and that the parameter determination procedure is very sensitive because of the high number of parameters, it is assumed that the results presented in Fig. 5 are average behavior. In consequence, both parameters q_1 and q_2 are taken constant and equal to the average of all the values shown in Fig. 5. Their respective values are 1.4 and 0.9.
The second step of the analysis consists in applying the mathematical procedure again with these values for q_1 and q_2. The purpose here is to check whether the fit between the mathematical function and the measured distributions is still good. This second run is conducted for the 5-parameter MEF distribution only. Comparisons are shown in Fig. 6.

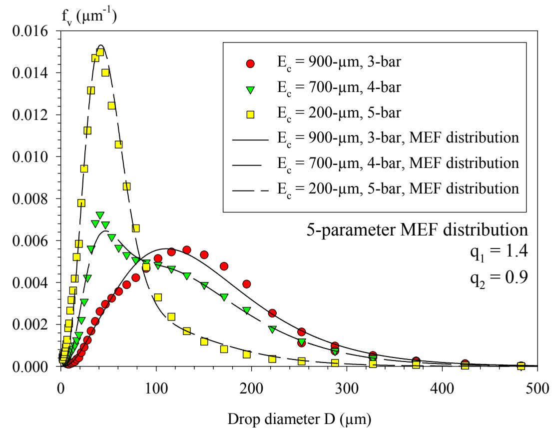

Fig. 6 Examples of comparison between mathematical and measured distributions (5-parameter, $q_1 = 1.4$, $q_2 = 0.9$)

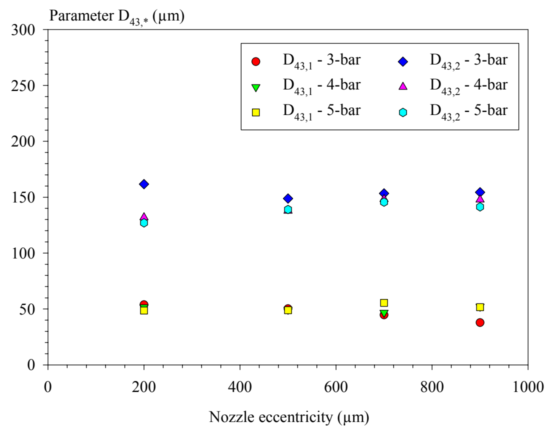

Fig. 7 Parameter $D_{43,1}$ and $D_{43,2}$ of the 5-parameter MEF distribution with $q_1 = 1.4$ and $q_2 = 0.9$.

It can be seen in Fig. 6 that the 5-parameter MEF distributions with constant values for q_1 and q_2 still offer a good representation of the drop-size distributions. This result is in agreement with behavior found in previous investigations [8-10]. The parameter q introduced by the mathematical drop-size distribution (Eq. (1)) characterizes mainly the disintegration process and is constant if the drops are produced the same way. This parameter is an intrinsic characteristic of the disintegration process. In the present study, one can say that the bimodal drop-size distributions result from the presence of two disintegration processes characterized by a parameter q equal to 1.4 and 0.9.

Figure 7 presents the value of the parameters $D_{43,1}$ and $D_{43,2}$ resulting from the second mathematical application. It is very much interesting to note that for the twelve situations investigated here, both parameters $D_{43,1}$ and $D_{43,2}$ are pretty independent of both the injection pressure and the nozzle eccentricity. This result is important. First, it confirms that the aerodynamic forces have a negligible influence on the two disintegration processes involved in the spray production. Indeed, one of the known effects of the aerodynamic forces is to reduce the characteristic length scale of a spray while increasing. Second, this result shows the importance of the parameter α in the present application. Indeed, the effects of the injection pressure and of the nozzle eccentricity are contained in this parameter.

Figure 8 shows the values of the blending parameter corresponding to the 5-parameter MEF

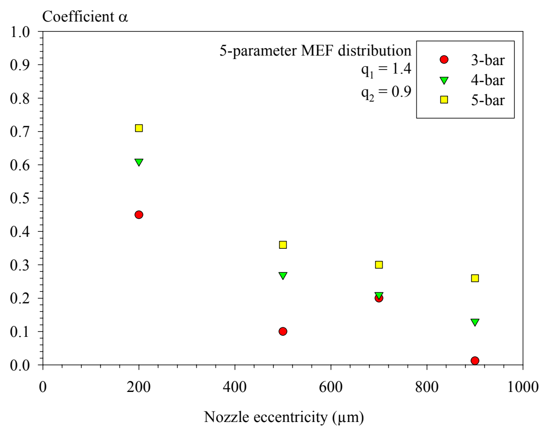

Fig. 8 Blending parameter α of the 5-parameter MEF distribution ($q_1 = 1.4$, $q_2 = 0.9$)

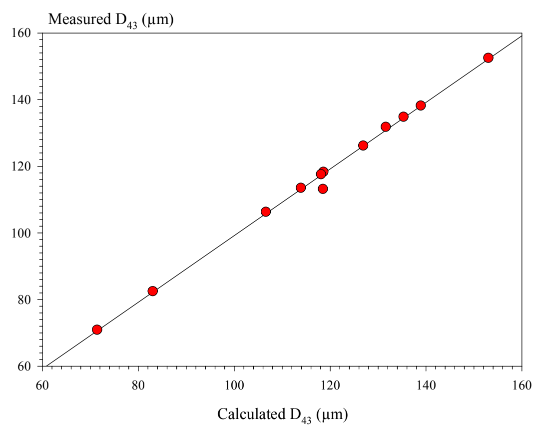

Fig. 9 Comparison between the calculated and the measured mean drop diameter D_{43}

application with $q_1 = 1.4$ and $q_2 = 0.9$. It appears that the parameter α depends on the injection pressure as well as on the nozzle eccentricity. Globally, it decreases when the eccentricity increases and when the injection pressure decreases. Above this, it is interesting to note that the evolution of the blending parameter with the injection pressure and with the nozzle geometry is rather organized. Thus, the development of a model to predict the drop-size distribution of the sprays produced by the simplified cavity nozzles investigated here should mainly concentrate on the determination of the blending parameter.

A previous application of the two-parameter Generalized Gamma function (Eq. (1) with $D_0 = 0$) on different sprays showed that the mathematical distribution conserved the mean diameter D_{43} [11]. In the present application, the mean diameter D_{43} of the mathematical distribution is given by:

$$D_{43} = \alpha D_{43,1} + (1-\alpha) D_{43,2} \tag{5}$$

Figure 9 compares the calculated and the measured mean diameter D_{43}. It can be seen that the agreement between the two diameters is very good. Thus, considering that the two parameters $D_{43,1}$ and $D_{43,2}$ are constant (see Fig. 7) and that the calculated D_{43} must be equal to the experimental D_{43}, Eq. (5) relates the parameter α with the measured D_{43} as shown in Fig. 10. In this figure the parameters $D_{43,1}$ and $D_{43,2}$ were given the average calculated from all the results presented in Fig. 7, namely, $D_{43,1}$ = 49-µm and $D_{43,2}$ = 145-µm. Thus, for the injectors considered in the present analysis, the drop-size distribution can be estimated from a unique information, the D_{43}. This diameter is used to evaluate the parameter α and the four other parameters are: $q_1 = 1.4$, $q_2 = 0.9$, $D_{43,1}$ = 49-µm and $D_{43,2}$ = 145-µm.
To validate this approach, it was tested on two other injectors with different dimensions as those used in the present analysis. The new injectors have the same disk 1 and the same disk 3 with a 200-µm eccentricity as the injectors used in the analysis (see Table 1) but differ by the thickness of the cavity disk (see Table 3). As for the previous injectors, measurements of the spray drop-size distribution were conducted at 50-mm from the injector at different injection pressures. For each measurement, the mean diameter D_{43} was used to determine the parameter α and the mathematical distribution was derived from the procedure described above.

	Disk 2 thickness (µm)	**Eccentricity (µm)**
Inj. A	125	200
Inj. B	200	200

Table 3 Geometrical dimensions of Inj. A and B

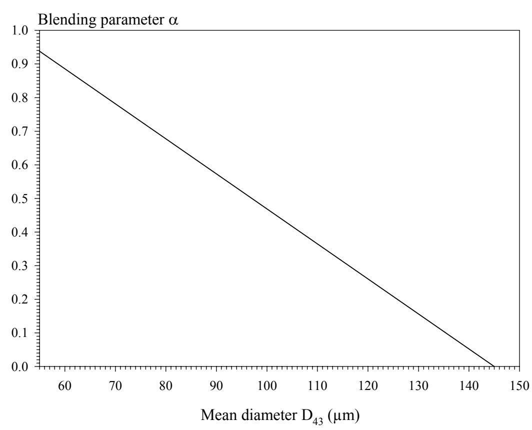

Fig. 10 Relation between the blending parameter s and the mean diameter D_{43}

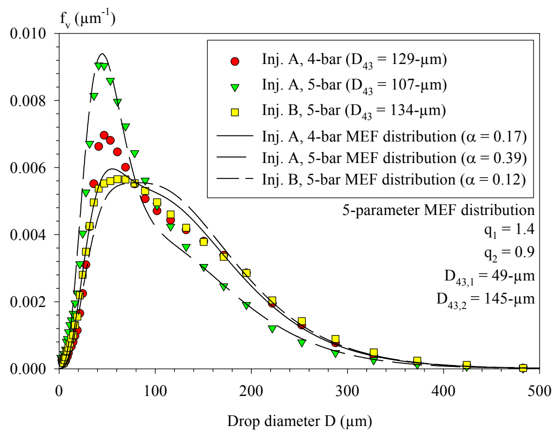

Fig. 11 Application to the MEF model to Inj. A and B

Figure 11 presents some comparisons between calculated and measured drop-size distributions. It can be seen in this figure that the agreement is rather good, indicating that a modification of the nozzle geometry does not modify the atomization process that appears to be mainly controlled by capillary effects.

5. Conclusion

The analysis, presented in this paper, shows that the Generalized Gamma function deduced from the application of the Maximum Entropy Formalism is very well adapted to represent bimodal drop-size distribution of sprays produced by cavity nozzle. It is interesting to note here that the application of this function on sprays produced by different injectors – including low and high-pressure swirl atomizer, ultrasonic atomizer, pintle-type low pressure gasoline injector, plain injector – led always to very good representation of the volume-based drop-size distribution. In the present application, the drop-size distributions were represented by a combination of Gamma Generalized function since the distribution is bimodal and five parameters were found sufficient to represent these distributions.

As far as the values of the parameters were concerned, we first obtained that the two parameters q (q_1 and q_2) could be taken constant whatever the injection pressure and the nozzle configuration. This behavior was already observed in previous applications [8-10]. It is an important result that suggests that the parameter q is an intrinsic characteristic of an atomization process and that its value is constant provided that the atomization process is unchanged. In the present situation, one can say that the bimodal drop-size distribution results from the presence of two disintegration processes, one being characterized by $q = 1.4$ and the other one by $q = 0.9$.

Two other parameters are the mean diameter D_{43} of each component of the global distribution. It is found that these two diameters are also independent of the injection pressure and of the nozzle configuration. This interesting result confirms that the aerodynamic forces due to the presence of the surrounding gaseous environment have negligible effects on the atomization processes present in the spray formation. It also shows that the influence of the injection pressure as well as of the nozzle geometry is mainly concentrated on the blending parameter α. This parameter is deeply related to the shape of the liquid issuing from the injector. Indeed, it was found in a previous study [1] that the shape of the issuing liquid flow is strongly dependent on the injection pressure and the nozzle geometry. Thus, the prediction of drop-size distributions produced by the cavity nozzles investigated here can be achieved by the determination of a single parameter. Thus, the problem of developing a model is simplified as it is shown here.

According to Lefebvre [12], one of the desirable attributes of mathematical expression that represent spray drop-size distribution is that they furnish some insight into the basic mechanism involved in atomization. It is clear that the mathematical distribution derived from the MEF fulfils this condition.

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