Development of a Spray Wall Impaction Model based on Drop Size Moments.

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The Eulerian-Eulerian approach proposed by Beck and Watkins [1] is used here to study the phenomena of a spray impinging on a wall. The basic fluid dynamics of an impinging spray can be simulated with this approach once the appropriate wall boundary conditions have been imposed. However, near the wall, the SMR and the velocity component normal to the wall are not simulated appropriately. Hence a compatible wall impaction model has been developed to take into account the different outcomes that a droplet can undergo after impaction on a wall (deposition, rebound and splash), and to correct the size and velocity distributions in the near wall region. This paper presents some corrections to the theory and results already presented by Lemini and Watkins [2]. It also presents the theory for obtaining a new droplet size distribution of the spray after impaction and some of the results obtained.

1. Introduction

The method of modelling sprays, using the moments of the droplet number size distribution, was first introduced by Beck and Watkins [3]. This method does not require the discretisation of droplets into size classes to capture the polydisperse nature of a spray flow. Instead, it solves both the liquid and the gaseous phases in an Eulerian manner [4]. In this way, the liquid phase is considered as a coherent whole and its properties are written in terms of the first four moments of its number size distribution function. The third and fourth moments are calculated using transport equations, and the other two are approximated from a presumed distribution function. Because the size distribution is different along the spray, it is truncated in order to fit the SMR obtained with the transport equations of the third and fourth moments.
This model is capable of predicting a basic wall spray, once the appropriate wall boundary conditions have been imposed. However, this does not take into account important effects that occur in a real spray, namely the deposition, rebound or break-up of droplets and the associated effects on the drop velocities. The modelling scheme of Beck and Watkins gives much information about the size distribution of the droplets in each computational cell. Hence it is possible to build a statistical wall impaction model capable of predicting the amount of liquid, within each near wall computational cell, that is involved in each one of the impaction regimes (i.e. deposition, rebound and splash). The effects on the number size distribution moments

of the liquid can then be calculated, along with the associated velocity components. The theory of this model, and some initial results, were presented by Lemini and Watkins [2]. Some corrections to the model are shown here. Also, the results of the droplet size distribution after the wall impingement are presented.

2. Mathematical Model

Table 1 summarises the wall impaction model presented by Lemini and Watkins [2]. It is important to note that this time the fractions of each moment experiencing splash, rebound or deposition are presented in terms of the spray size distribution proposed by Beck and Watkins [3] as:

$$n(r) = \frac{16r}{r_{32,ref}^{2}} e^{\frac{-4r}{r_{32,ref}}} \tag{1}$$

In this way, the exact amount of liquid volume, the droplet's surface area and the number of droplets depositing, rebounding or splashing on the wall can be accurately calculated.

Dimensionless number for impingement regimen criteria:	$k = \left(\frac{\rho^3 d^3 u^5}{\sigma^2 \mu}\right)^{0.25}$	[5]	(2)
If $k \leq k_d$	Deposition k_d=15.0	(guess)	
If $k_d < k < k_s$	Rebound		
If $k \geq k_s$	Splash k_s=57.7	(according to [5])	
Depositing fraction:	$SQ_i^d = Q_0 \int_0^{r_d} n(r) \cdot r^i dr$		(3)
Rebounding fraction:	$SQ_i^r = Q_0 \int_{r_d}^{r_s} n(r) \cdot r^i dr$		(4)
Splashing fraction:	$SQ_i^s = Q_0 \int_{r_s}^{\infty} n(r) \cdot r^i dr = Q_0 (1 - SQ_i^d - SQ_i^r)$		(5)
Deposition/Splashing reference radius:	$r_{d/s} = \left(\frac{k_{d/s}^4 \sigma^2 \mu}{8u^5 \rho^3}\right)^{\frac{1}{3}}$		(6)
Source terms from splashing:	$J_i(u) = Q_0 \int_{k_S}^{\infty} \int_0^1 r_{in}(k,u)^i d(k) g(k) m(k,r^*) r^{*i} dr^* dk$		(7)
Where,	$d(k) = \frac{1}{\int_0^1 m(k,r^*) r^* dr^*}$		(8)
	$g(k) = \frac{16}{3r_{32}^2} \left(\frac{k^5 \sigma^4 \mu^2}{\rho^6 u^{10}}\right)^{\frac{1}{3}} e^{\frac{-2}{r_{32}}\left(\frac{k^4 \sigma^2 \mu}{\rho^3 u^5}\right)^{\frac{1}{3}}}$		(9)
	$m(k,r^*) = \frac{r^*}{a} e^{-\left(\frac{r^*}{b}\right)^{\alpha}}$		(10)
	$a = 1.96602k^2 - 9.1349\times10^{-2} k + 10.419257$ $b = -5.3313\times10^{-5} k^2 + 1.53062\times10^{-2} k - 0.936192$ $\alpha = -1.05522\times10^{-3} k^2 + 0.34599k - 26.130703$		(11)
The final quantity of the i^{th} moment Q_i is given by:	$Q_i^{new} = Q_i^{old} - SQ_i^d - SQ_i^s + J_i(u)$		(12)

Table 1. Spray wall impaction model equations regarding the moments of the spray.

Note also that the function *d(k),* which describes the number of secondary droplets outgoing from a single splashing droplet, is only a function of the *k* number; and not a function of *k* and of the impinging velocity as it was presented in [2]. Figure 1 shows the corrected shape of *d(k)*.

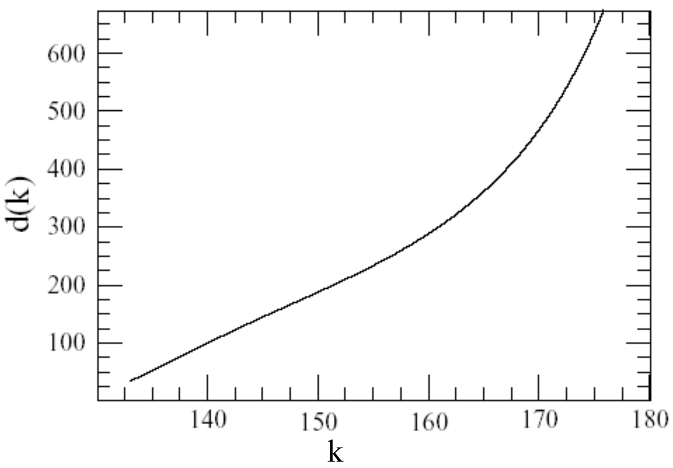

Figure 1. Number of secondary droplets as a function of *k.*

The Sauter Mean Radius of the spray within a cell is given by,

$$SMR = \frac{\int n(r) r^3 dr}{\int n(r) r^2 dr} = \frac{Q_3}{Q_2} \tag{13}$$

With the model presented in Table 1, one does not have to determine the complete size distribution of the spray to calculate the SMR. One would only need to calculate the new values of the corresponding moments in order to find the new value of the SMR. This can be done using equation (12).

Nevertheless, if one needs more information, it is also possible to find the complete size distribution of the resulting spray after impinging on a wall. This work reports a general way to do this and presents a specific solution for a polydisperse spray, which initially (i.e. before impacting the wall) has a probability size distribution described by equation (1). The $r_{32,ref}$ for the presented experiments has been taken as $2.5\text{x}10^{-5}$ m, which is a typical value of a droplet radius in a diesel engine spray. Also, this value is used by Beck in some of the calculations reported in [1].

Considering that all the liquid within a near wall computational cell impacts the wall, the size distribution of the outgoing spray can be defined as:

$$n'(r) = \frac{\gamma}{\gamma^2 + \beta^2} n_{reb}(r) + \frac{\beta}{\gamma^2 + \beta^2} s(r) \tag{14}$$

where,

$$\gamma = \int_{r_d}^{r_s} n_{reb}(r) dr \tag{15}$$

and,

$$\beta = \int_0^\infty s(r) dr \tag{16}$$

The first term in equation (14) is the size distribution of the fraction of droplets that rebound on the wall. This term is defined as:

$$n_{reb}(r) = \begin{cases} n(r) & r_d \le r \le r_s \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

Note that r_d and r_s, defined by equation (6), are functions of the impingement velocity. Hence, for a given liquid, the amount of spray that rebounds on the wall and its size distribution is given by the truncation of equation (1) as a function of the impingement velocity.

The second term refers to the size distribution resulting from the splashing droplets. This term is defined as:

$$s(r) = \int_{k_s}^{\infty} d(k)g(k)m(k,r)dk \tag{18}$$

This is not a probability size distribution itself. *g(k)* is the probability that a droplet with a certain *k* exists in the cell before impaction, hence it is dimensionless. *m(k,r)* is the probability that an outgoing secondary droplet with an specific radius *r* will form from an incoming droplet with a given *k* number. This number is also dimensionless. *d(k)* is not dimensionless, it represents the number of secondary droplets resulting from an impinging droplet with a certain *k* number. Hence, this distribution itself does not give clear information of what is happening during splashing. Nevertheless, if it is multiplied by the initial number of droplets in the cell, it will be automatically converted into a number size distribution. Now, to obtain a probability size distribution, it should be divided by the new number of droplets in the cell. All these steps are summarized by the factors defined by equations (15) and (16), which also contain the fraction of droplets that rebounded on the wall.

Figure 2 shows both components of the new distribution and the one resulting from combining them. These results have been normalized with respect to the reference Sauter Mean Radius $r_{32,ref}$,

$$r' = \frac{r}{r_{32,ref}} \tag{19}$$

Note that in this figure *s(r')* is shown because its shape is exactly the same that the corresponding probability density function would follow. Also, it gives information about the range of sizes obtained from the splashing fraction.

One can see that, with the current initial distribution, for velocities lower than 3.0 m/s the splashing contribution is not important compared to the rebounding one. From this distribution one can also see the importance of the deposition regime at this velocity, i.e. the truncation limit r_d is located almost at the tail of the distribution.

At 6.0 m/s the splashing starts to become noticeable, but rebounding is still much more important. r_d is moved to the left in relation to the one obtained for 3.0 m/s. It was expected that as the impingement velocity increases the deposited amount of liquid decreases.

At 9.0 m/s splashing is very important, but a couple of small discontinuities due to the rebounding contribution are still noticeable. Note also that not only r_d has been moved to the left, but also r_s. It was also expected to find a decrement in the rebounded fraction as the impingement velocity increases.

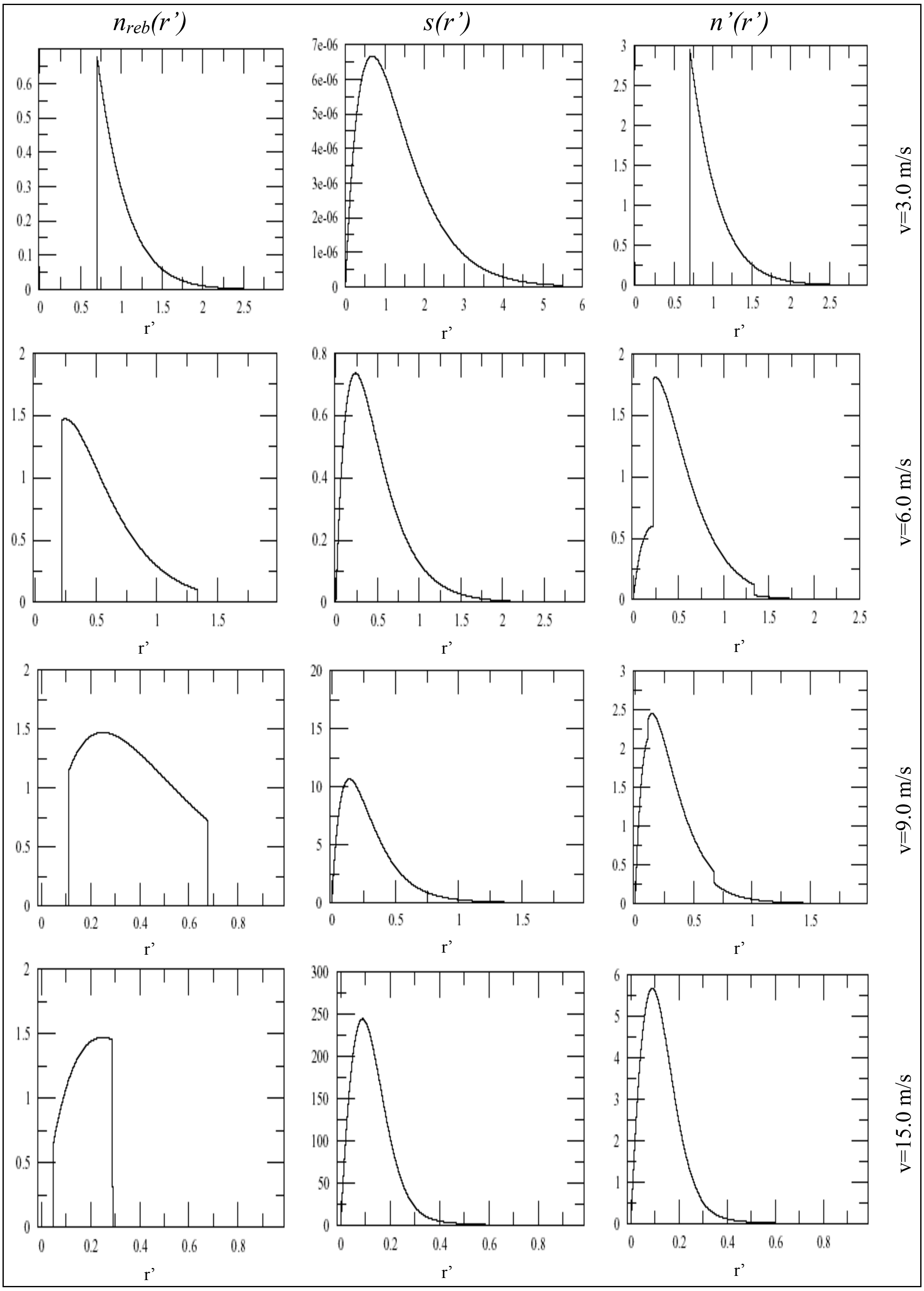

Figure 2. Probability size distribution of a wall spray after impaction at different incoming velocities and its components $n_{reb}(r')$ and $s(r')$.

At 15.0 m/s one can no longer see any discontinuities in the values of the distribution, although there is some rebound, it is very small and could be neglected.
It is interesting to note that the mean values of the new distribution are located very near the lower limit of the rebounding distribution and that the variance of the new distribution is much smaller than that corresponding to the original distribution. This means that, after impaction, the spray ends up with a more monodisperse distribution in the near wall region. Figure 3 compares the distributions before and after impaction of a spray with a velocity equals 15.0m/s.

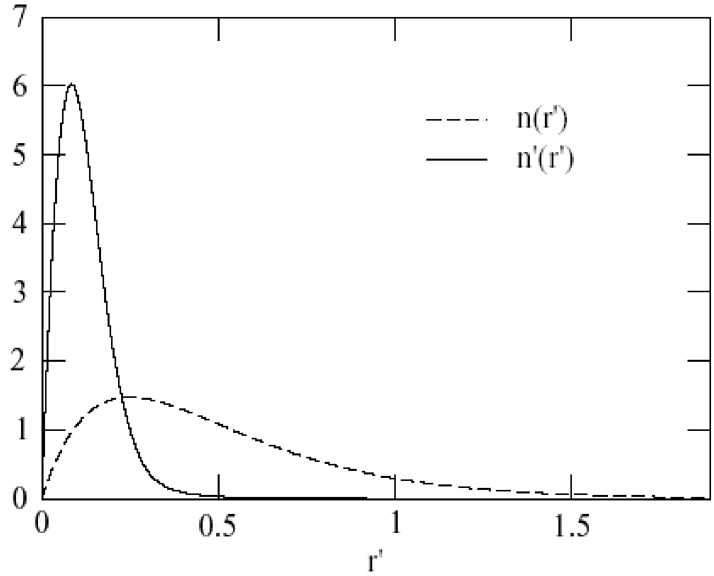

Figure 3. Comparison between the size distribution of the spray within a cell before and after impaction. Impaction velocity equals 15m/s.

Figure 4 compares the size distributions between the secondary droplets coming from a single splashing drop and from a spray with a velocity equals 15.0 m/s. The first distribution is that obtained for an incoming droplet with a *k* value equals 152; the same value corresponds to the second distribution if one calculates the average *k* number of the droplets traveling at a velocity of 15.0 m/s. This average *k* is calculated as follows,

$$k = \left(\frac{8\rho^3 r_{32}^3 u^5}{\sigma^2 \mu} \right)^{.25} \qquad r_{32} = r_{32,ref} \tag{20}$$

One would expect that both distributions were similar because of the small importance that the rebounding part has for this case. Figure 4 compares the magnitude and shape of the distribution calculated with the model, and gives an idea of its accuracy. The compared distributions were not expected to be equal.

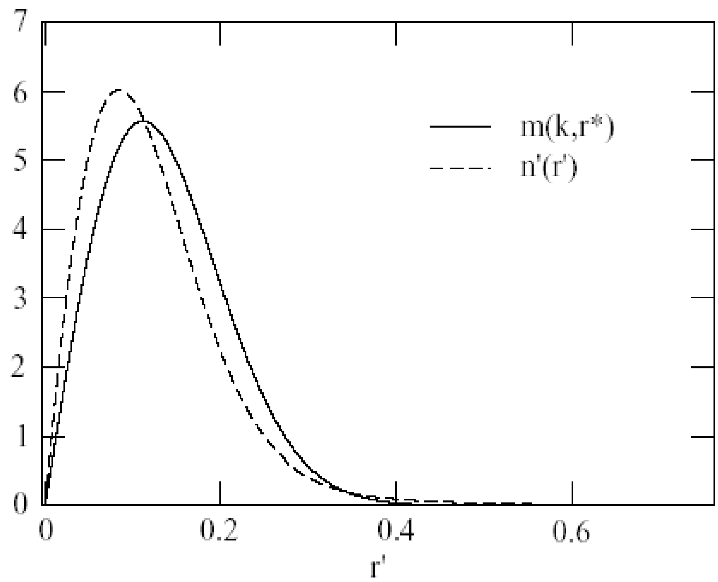

Figure 4. Comparison between the size distribution of secondary droplets outgoing from a single droplet and the size distribution of a spray with the same mean *k* number.

3. Conclusions

The present model is capable of delivering much more information than previous wall impaction models due to its statistical nature. A complete new size distribution after impaction can be calculated with a much smaller amount of computational time than the one needed to do the same with a DDM approach.

It is a fact that the more experimental information one has about the behavior of the splashing regime the more accurate this type of modeling will become. Presently, in order to obtain the results presented here, many assumptions have been done due to the small amount of statistical data (i.e. k_d=15.0, no information about the secondary droplet distribution for k values smaller than 133.2 and larger than 186.6). Nevertheless, the distributions have presented very sensible trends as can be confirmed in Figure 4.

Nomenclature

Functions

d(k)	*Number of secondary droplets from a single splashing droplet with a certain k value.*
g(k)	*Probability function of the present droplets with a certain k value.*
$J_i(u)$	*Source terms for the* i^{th} *moment from the splashing droplets.*
m(k,r')	*Size distribution of secondary droplets as a function of the k value of the incoming droplet.*
n(r)	*Size distribution of the spray before impacting a wall.*
n'(r)	*Size distribution of the spray after impaction a wall.*
s(r)	*Information about size distribution of secondary droplets from splashing.*

Variables	
d	*Diameter*
k	*Dimensionless number for impingement regimen criteria.*
Q_i	i^{th} *moment of the spray.*
r	*Radius.*
r'	*Normalized radius with respect to the reference SMR.*
r^*	*Normalized radius with respect to the original splashing droplet radius.*
SMR	*Sauter Mean Radius.*
Greek symbols	
μ	*Liquid viscosity.*
ρ	*Density.*
σ	*Surface tension.*
Subscripts	
d	*Deposition.*
reb	*Rebound.*
s	*Splash.*
32	*Sauter Mean .*
32,ref	*Reference Sauter Mean.*

References

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