Heat transfer during single drop impact onto a wetted surface

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In the present work the heat transfer during impact of single drop with high Weber number onto a liquid film is modelled. The temperature fields inside the inner region, the outer film and in the crown are calculated. The time evolution of the rim temperature and of the secondary droplets temperature (in the case of splash) is calculated.

1. Introduction

Spray cooling is important in many industrial applications, including steel rolling industry, electronics and cooling of human tissues in medicine. Due to the extreme complexity of the phenomenon, the mechanisms of the spray cooling are not yet fully understood. There exists a very scarce amount of literature devoted to the theoretical investigation of heat transfer by spray cooling [1], [2]. In most of the existing models, the effect of the individual drop impacts on the overall heat transfer characteristics is not analyzed.

Spray impact onto a substrate immediately creates a relatively thin, irregular, time-varying liquid film. This means that one of the important phenomena of spray cooling is the impact of a single drop onto a wetted surface and its influence on the local heat transfer. The

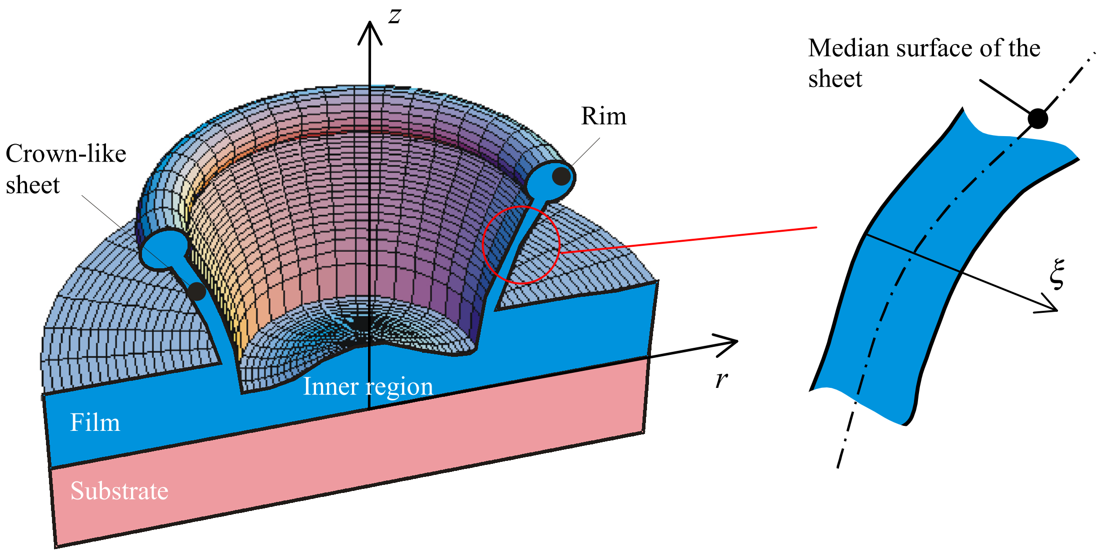

Fig. 1: Sketch of single drop impact onto a liquid layer

present work is aimed at theoretical description of heat transfer during impact of a single drop onto a liquid film (Fig. 1). These results can be considered a first step towards modeling of spray cooling.

2. Hydrodynamics of drop impact

The hydrodynamics of single drop impact onto a liquid film have been studied in the experimental investigations of [3]-[5]. Recent theoretical models [6]-[7] describing the hydrodynamics of splashing drops are based on the assumption that the inertia dominates viscosity and surface tension, which is justified for large impact Reynolds and Weber numbers. The liquid film on the substrate is assumed to consist of two main regions: an inner region with radially expanding flow produced by a drop impact, and an outer undisturbed liquid film. The velocity field in the inner film and its thickness are [6]

$$V_l(r,t) = \frac{r}{t+\tau}, \qquad h_l(t) = \frac{\eta}{(t+\tau)^2}, \tag{1}$$

where V_l is the radial velocity in the inner part of the film (lamella) and h_l is its thickness, r is the radial coordinate, t is the time, and τ and η are constants. Here and below all the variables are given in the dimensionless form with the initial drop diameter D_0 being used as the length scale and the impact velocity U_0 as the velocity scale.

The boundary of the radius R_B between the inner expanding and the outer undisturbed regions of the film is the base of the crown-like sheet ejected during the splash. It was shown in [6] that in the case of the impact onto a stationary, uniform liquid film the propagation of the base of the crown follows the square-root law:

$$R_B(t) = \beta\,(t+\tau)^{1/2}, \tag{2}$$

where β is a constant.

An analytical expression for the shape of the median surface of the crown-like sheet and its thickness has been obtained in [7] for the cases of high Reynolds and high Weber numbers, when the viscous drag and the effect of capillary forces were neglected. This shape of the median surface is determined in parametric form $\boldsymbol{x} = \boldsymbol{X}_J(t_B,t) = R_J(t_B,t)\boldsymbol{e}_r + Z_J(t_B,t)\boldsymbol{e}_z$ with the radius and the height being

$$R_J(t_B,t) = R_B(t_B)\left[1 + \frac{h_l(t_B)}{h_l(t_B)+h_f}\,\frac{t-t_B}{t_B+\tau}\right] \tag{3a}$$

$$Z_J(t_B,t) = R_B(t_B)\frac{h_l^{1/2}(t_B)\,h_f^{1/2}}{h_l(t_B)+h_f}\,\frac{t-t_B}{t_B+\tau} - \frac{(t-t_B)^2}{2Fr} \tag{3b}$$

where h_f is the dimensionless thickness of the undisturbed film $Fr = U_0^2/(g\,D_0)$, is the Froude number, g is the gravitational acceleration, and $t_B < t$ is a parameter. A material point ejected with the crown from the wall at the time instant t_B is located at $\boldsymbol{x} = \boldsymbol{X}_J(t_B,t)$ at the time instant t.

The thickness of the crown, obtained from the mass balance takes the form:

$$h_J(t_B,t) = \left[h_l(t_B)+h_f\right]\frac{R_B(t_B)}{R_J(t_B,t)}\,\frac{\sqrt{G_1^2(t_B,t_B)+G_2^2(t_B,t_B)}}{\sqrt{G_1^2(t_B,t)+G_2^2(t_B,t)}} \tag{4}$$

where the functions G_1 and G_2 are defined as

$$G_1(t_B,t) = \frac{1}{2(t_B+\tau)^{3/2}}\left[t_B+\tau+\frac{4h_l^2(t_B)}{(h_l(t_B)+h_f)^2}(t-t_B)+\frac{h_l(t_B)}{h_l(t_B)+h_f}(3t_B-5t-2\tau)\right]$$

$$G_2(t_B,t) = \frac{t-t_B}{\beta Fr} + h_f^{1/2} h_l^{1/2}(t_B) \frac{h_l\,(t_B)(t-2\tau-3t_B) - h_f\,(3t+2\tau-t_B)}{(t_B+\tau)^{3/2}\left[h_l(t_B)+h_f\right]^2}$$

The velocity of the liquid in the crown is

$$\boldsymbol{V}_J(t_B,t) = \boldsymbol{V}_B(t_B) - Fr^{-1}(t-t_B)\boldsymbol{e}_z = \frac{\beta[h_l(t_B)\boldsymbol{e}_r + h_l^{1/2}(t_B)h_f^{1/2}\boldsymbol{e}_z]}{\left[h_l(t_B)+h_f\right](t_B+\tau)^{1/2}} - Fr^{-1}(t-t_B)\boldsymbol{e}_z$$

where $\boldsymbol{V}_B(t)$ is the velocity of the ejected liquid at the base of the crown (on the substrate). See [7] for more details.

The ejected liquid sheet is bounded by a rim. This rim is formed due to capillary forces. The mass and momentum balance of this rim yields the following expression for the relative to the sheet velocity [8]

$$U_R = \sqrt{\frac{2}{We\,h_J(t_R,t)}} \tag{5}$$

where $We = \rho D_0 U_0^2/\sigma$ is the Weber number, ρ and σ are the liquid density and surface tension, $h_J(t_R,t)$ is the sheet thickness at the rim, $t_R(t)$ is a parameter. The material point ejected from the wall at the time instant t_R reaches the rim at the time instant t.

The rim location is defined as $X_R(t) = X_J(t_R,t)$ where the parameter t_R is calculated by numerical integration of the ordinary differential equation obtained in [7] from equation (5) and from the mass balance of the crown:

$$\frac{dt_R(t)}{\mathrm{d}t} = \sqrt{\frac{8}{We}}\,\frac{R_J(t_R,t)\,h_J^{1/2}(t_R,t)}{\beta^2[h_l(t_R)+h_f]} \tag{6}$$

If there is no splash, the volume W_R of the rim equal to the volume of the liquid ejected from the wall before t_R:

$$W_R(t) = 2\pi\int_0^{t_R} R_B(t)Q(t)dt \tag{7}$$

where the flux Q of the ejected liquid is

$$Q(t) = \frac{1}{2}[h_l(t)+h_f]\,V_l(R_B,t) \tag{8}$$

Note, that the remote asymptotic solution (1) is valid only at large values of time t. In the present analysis we neglect the volume of the liquid collected in the rim during the initial phase of drop deformation $t_1 \sim 1$. The expression for the rim volume obtained with the help of (7) and (8) is

$$W_R(t) = \pi\beta^2(t_R - t_1)\left[h_l^{1/2}(t_1)\,h_l^{1/2}(t_R) + h_f\right] \tag{9}$$

2. Model of heat transfer during drop impact

In previous numerical studies, the heat transfer during drop impact onto a dry surface has been considered [10], [11]. The heat transfer during drop impact onto a wetted surface has not been previously studied.

In the present model we consider the impact of a cold drop onto a quiescent and smooth thin film covering a thin hot substrate. The substrate is heated by a constant heat flux, q_w^*. Since the substrate is thin, the heat conduction in the substrate parallel to the wall plane may be neglected. Therefore, it is assumed that the condition of the constant heat flux is applied at the solid-liquid interface. Before the drop impact, the temperature in the liquid film varies linearly with the height above the wall. The wall temperature is determined by the thermal conductivity of the liquid, k, the liquid film thickness, the gas temperature and the heat transfer coefficient at the liquid-gas interface. The temperature field in the film in the outer region of impact ($r > R_B$) remains unchanged after the drop impact. The temperature field in the liquid film in the inner region ($r < R_B$) changes due to the radially expanding film produced by the drop impact. The temperature field inside the crown-like sheet is determined by the conditions at the crown base and by the heat transfer between the liquid and the gas. The crown-like sheet is bounded by a free rim. The rim temperature is determined by the liquid inflow from the crown. If the values of the Weber and Reynolds numbers are high enough, free rim jetting and splashing occurs, leading to ejection of secondary droplets from the rim. The temperature of the secondary droplets is determined by the crown temperature adjacent to the rim.

In the following we analyze the temperature fields inside the liquid film adjacent to the wall and in the free film forming the crown for times exceeding D_0/U_0, or t>1. Since the films are very thin, the heat conduction parallel to the film is neglected both for the wall film and the free film. We assume that the drop impact takes place in the saturated vapor environment, so that the heat transfer between the liquid and the gas (vapor) takes place by evaporation, and the temperature at the liquid-vapor interface is equal to the liquid saturation temperature.

Define the dimensionless liquid temperature in the following form:

$$T = \frac{T^* - T_s^*}{T_{w0}^* - T_s^*} , \tag{10}$$

where T^* is the dimensional liquid temperature, T_s^* is the saturation temperature, and T_{w0}^* is the wall temperature before the drop impact, which is equal to

$$T_{w0}^* = T_s^* + \frac{q_w^* h_f D_0}{k} . \tag{11}$$

After the drop impact the linear temperature distribution in the film in the outer region remains unchanged,

$$T(r > R_B, z, t) = 1 - z/h_f , \tag{12}$$

where z is a dimensionless coordinate normal to the substrate.

The temperature field in the inner region (inside the expanding film) is non-stationary. This expanding film is so thin that the characteristic time of the heat conduction through it is much smaller than the characteristic time of the film expansion. Therefore, the temperature field in the inner region is quasi-stationary, and is determined by the instantaneous film thickness given in Eq. (1). The liquid temperature in the inner region is given by

$$T(r < R_B, z, t) = \frac{h_l(t) - z}{h_f} = \frac{1}{h_f}\left[\frac{\eta}{(t+\tau)^2} - z\right]. \tag{13}$$

As it is seen from Eq. (13), the temperature field in the inner region depends on time and on the vertical coordinate, but is independent of the radial coordinate, r.

At $r = R_B$ a kinematic discontinuity exists, leading to an ejection of a crown-like sheet, described in [7]. Since normally $h_l << h_f$, the temperature distribution in the free film at the crown base is determined by the temperature field in the outer film. The film is separated from the wall. Consider a ring-shaped film element, which has reached the rim at the time instant t_C. This element has been separated from the wall at the time instant $t_B = t_R(t_C)$, where the function t_R is determined by the solution of differential equation (6). The time spent by the ring element in the crown is equal to $t_C - t_B$. During this time the ring element is thinned (see Eq. 4.) and elongated, so that its mass stays constant. Initially, it has a linear temperature distribution, corresponding to its wall-attached value. During the time spent in the crown, this element is cooled by evaporation. Consider the non-stationary heat conduction in the ring element. The characteristic dimensionless time of heat conduction in a film of thickness h_J is $t_{TH} = h_J^2/A$, where $A = k/(\rho c U_0 D_0)$ is the dimensionless thermal diffusivity of the liquid, and c is the liquid specific heat. For typical range of the drop impact parameters $t_{TH} >> t_C - t_B$, which means that during the time spent in the crown the temperature is changed only in a small layer adjacent to the initially hot surface. Therefore, the temperature field in the ring element is given by

$$T(t_B, \xi, t) = \mathrm{erf}\left[\frac{\xi}{2\sqrt{A(t-t_B)}}\right] - \frac{\xi}{h_J(t_B, t)} \;, \tag{14}$$

where erf denotes an error-function, and the coordinate ξ is measured normal to the crown median surface, starting from the inner crown surface, $0 \le \xi \le h_J(t_B, t)$.

To determine the temperatures of the rim and of the secondary droplets, we need the average crown temperature at the location where it enters the rim. This is given by the following:

$$\overline{T}_{CR}(t) = \frac{1}{h_J[t_R(t), t]} \int_0^{h_J[t_R(t), t]} T[t_R(t), \xi, t]\, d\xi \approx \frac{1}{2} - \frac{2}{h_J[t_R(t), t]} \sqrt{\frac{A \cdot [t - t_R(t)]}{\pi}} \;. \tag{15}$$

When an element of the crown enters a rim with a relative velocity given by Eq.(5), it is mixed by a roll convective motion with the liquid inside the rim. Since this mixing is very intensive, the temperature inside the rim is approximately homogeneous (rapid mixing hypothesis). The heat transfer by evaporation plays a minor role in determination of the rim temperature. Therefore, the rim temperature in the absence of splash can be calculated by integration:

$$T_R(t) = \frac{2\pi}{W_R(t)} \int_{t_1}^{t} \overline{T}_{CR}(\tilde{t}) \cdot h_J[t_R(\tilde{t}), \tilde{t}] \cdot R_J[t_R(\tilde{t}), \tilde{t}] \cdot U_R(\tilde{t})\, d\tilde{t} \;. \tag{16}$$

In the case of splash the secondary droplets leave the crown with the temperature equal to $\overline{T}_{CR}(t)$.

3. Results and discussion

The main mechanisms of the substrate cooling due to the drop impact at times exceeding unity are the reduction of the wall temperature in the inner region as a result of the film thinning, and the evaporative cooling of the film separated from the wall (the crown-shaped sheet).

The time evolution of the dimensionless wall temperature in the inner region is depicted in Fig. 2. The drop diameter, D_0, is equal to 500 μm, the dimensional film thickness in the outer region is kept constant at a value of 100 μm, and the impact velocity varies from 4 m/s to 12 m/s. The Weber number varies from 112 to 1010. The wall temperature rapidly decreases with time. Simultaneously, the area of this "cold" inner region increases. It is seen that the wall dimensionless temperature decreases with increasing the impact velocity. At the same time, the area of the inner region increases with increasing the impact velocity. This means that, for the given drop and film geometry, the cooling is more efficient for larger impact velocities. However, when the films becomes very thin, a film dryout may take place, which drastically deteriorates the heat transfer. This issue should be addressed in the future.

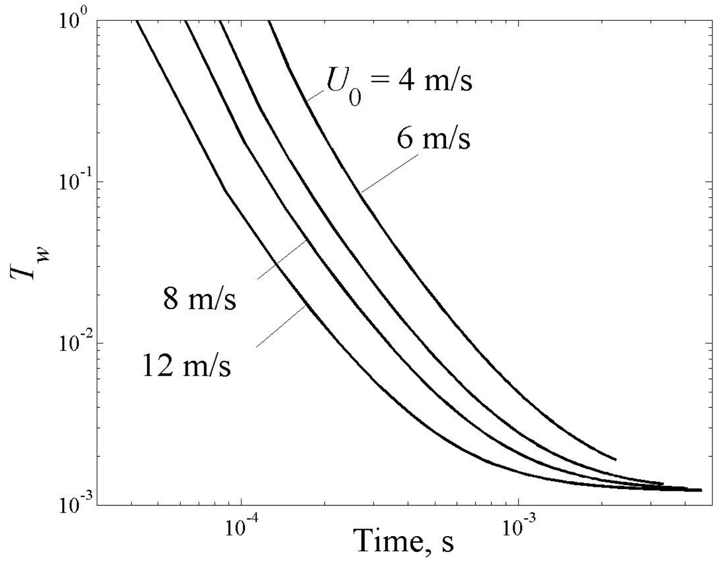

Fig. 2. Temporal dependence of the dimensionless wall temperature in the inner region for various impact speeds (D_0 = 500 μm, h_f=0.2)

Furthermore, the modeling of the hydrodynamics in the inner region can be improved by accounting for the effects of viscosity.

Figure 3 depicts the rim temperature for an impact of a drop with $D_0 = 200$ μm, $U_0 = 10$ m/s, $h_f = 0.29$, $We = 280$. Two different types of impact are possible with these impact parameters depending on the Reynolds number, namely, deposition (if the impact velocity is below the splash threshold) or splash. These two possibilities are considered in Fig. 3: in the first one no splashing occurs, and the rim keeps its torroidal shape until it falls onto the film. The temperature of the rim slowly changes with the time due to the mixing with the liquid inflowing from the crown. In the second case the rim breaks-up into small jets which further disintegrate to form secondary droplets. The initial temperature of these droplets is equal to the average temperature of the crown-rim interface, $\overline{T}_{CR}$. At the

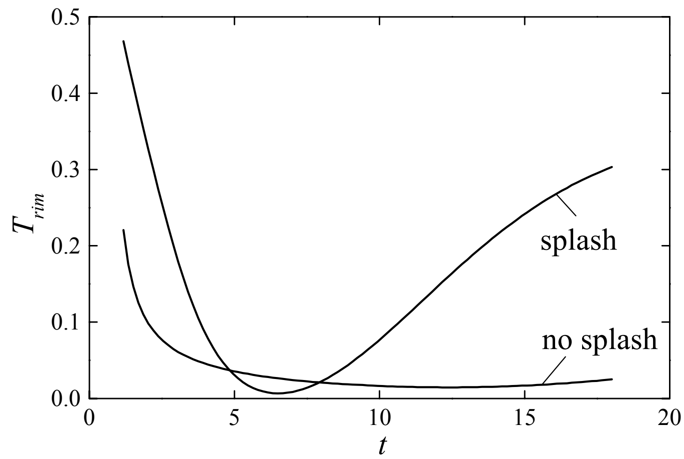

Fig. 3. Rim temperature as a function of time.

beginning this temperature decreases with time, since the time spent by the liquid in the crown increases, and, therefore, the heat flow by evaporation increases. For $t > 6$ the temperature of the secondary droplets increases. The temperature of the secondary droplets strongly depends on the time moment of their emergence from the crown.

Figure 4 presents the temperature field for an impact of a drop with the same parameters as in Fig. 3 for six different time instants between $t = 2$ and $t = 15$, a short time before the rim falls onto the film. The inner film is very thin, and it cannot be seen at the given scale except for the first picture ($t = 2$).

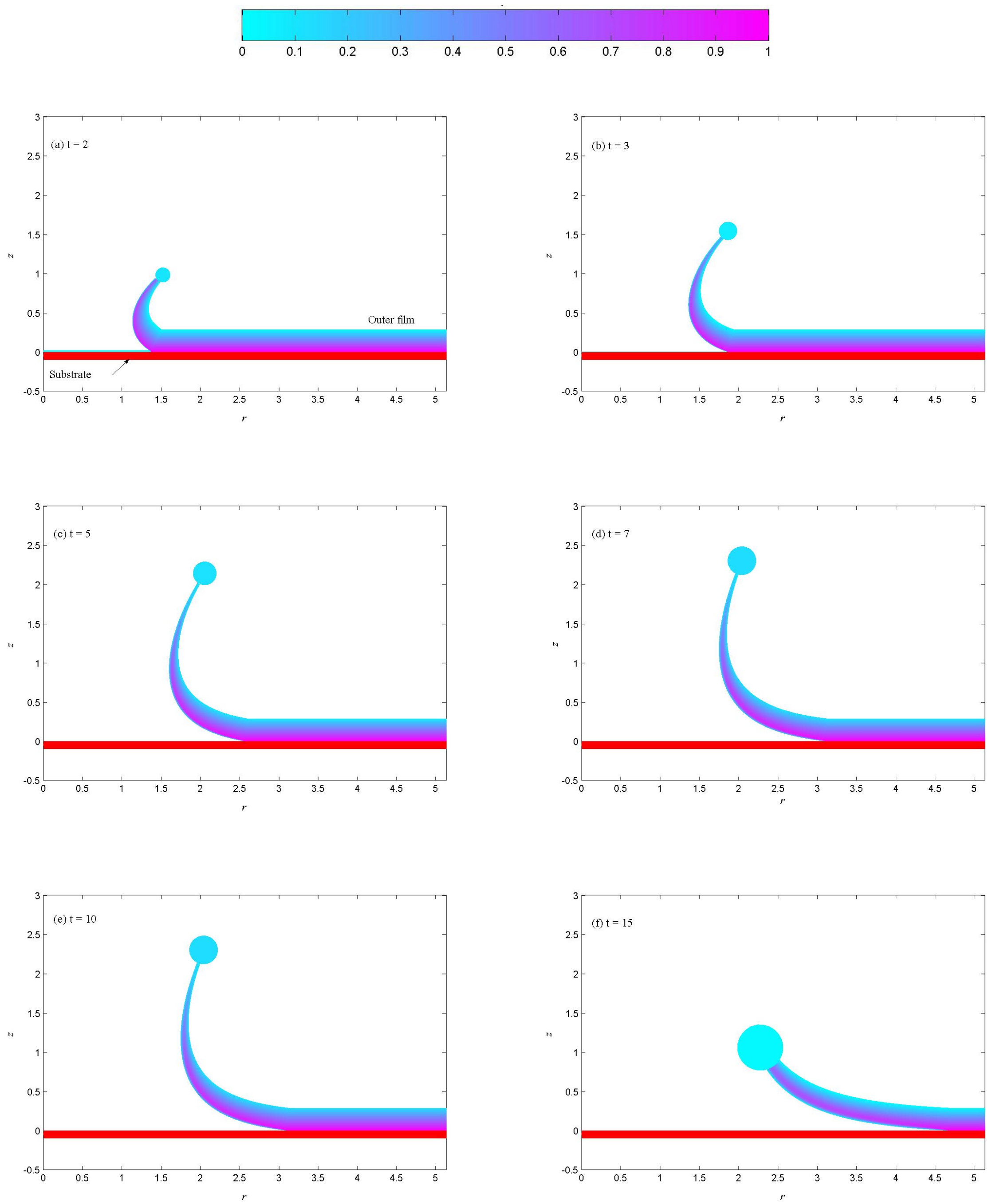

Fig. 4. The dimensionless temperature distribution in the crown-like sheet and in the outer liquid film at various time instants.

4. Conclusions

A model of heat transfer during a drop impact onto a liquid film is developed for large Weber and Reynolds numbers. This model describes the temperature distribution in the outer film, the non-stationary heat transfer in the expanding liquid film in the inner region, as well as the non-stationary heat transfer in the crown-like sheet. The liquid-vapor interface is kept at the saturation temperature. The basic heat transfer mechanisms in different regions of the fluid are identified.

This model enables calculation of the time-dependent substrate wall temperature, as well as the temperature of the rim and the secondary droplets. It can be used for comparison of the cooling efficiency of different drop impact regimes. In particular, it has been shown that increasing impact velocity leads to decreasing of the substrate wall temperature in the inner region. It has also been shown that the crown rim temperature depends on the impact regime (deposition or splash).

In the future, the present heat transfer model will be extended to include the effects of the oblique drop impact (drop impact on a moving film), drop interaction and the film fluctuations produced by spray impact. These enhancements will form a basis for a refined model of spray cooling.

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References

[1] Hodgson J W, Sunderland J E 1968 *I&EC Fundamentals* **7** 567-572.

[2] Graham K M, Ramadhyani S 1996 *ASME J. of Heat Transfer* **118** 343 – 349.

[3] Levin Z, Hobbs P V 1971 *Phil. Trans. R. Soc. Lond. A* **269** 555–585.

[4] Macklin W C and Metaxas G J 1976 *J. Appl. Phys.* **47** 3963–3970.

[5] Cossali G E, Coghe A and Marengo M 1997 *Experiments in Fluids* **22** 463–472.

[6] Yarin A L and Weiss D A 1995 *J. Fluid Mech.* **283** 141–173.

[7] Roisman I V, Tropea C 2002 *J. Fluid Mech*. **472** 373 – 397.

[8] Taylor G I 1959 *Proc. R. Soc. London Ser*. A **253** 296 – 312.

[9] Roisman I V, Tropea C 2002 *Proc. 18th Ann. Conf. on Liquid Atomization and Spray Systems ILASS-Europe,* 9-11 September, Zaragoza.

[10] Healy W M, Hartley J G, Abdel-Khalik S I 2001 *Int. J. Heat Mass Transfer* **44** 3869-3881.

[11] Pasandideh-Fard M, Aziz S D, Chandra S, Mostaghimi J 2001 *Int. J. Heat and Fluid Flow* **22** 201-210.