

# Concentration fields in evaporating droplets

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This paper presents an analytical solution of the diffusion equation on a spherical domain with its surface shrinking linearly with time. The solution is given as a series expansion in confluent hypergeometric functions and is valid for arbitrary ratios of rate of shrinkage of the surface of the sphere to diffusion coefficient in the liquid phase. The description derived allows for a prediction of the morphology of the dried particles, i.e. whether hollow or solid spheres of the dried substance result from the drying process. The field of application of the results is spray drying of solutions of solid substances with very low vapor pressure, e.g. salt solutions.

## 1. Introduction

Technical processes like spray drying and air/fuel mixture formation rely on the evaporation of liquid components from droplets in a hot gaseous environment. In the former process, the dry solute is the goal of the process, while in the latter the gaseous mixture of air and fuel vapor is desired. In both cases, the droplet liquid undergoes a temporal change in composition due to the liquid evaporation, which may lead to the formation of considerable concentration gradients inside the droplet. It has been recognized quite some time ago that in spray drying the mass transfer processes inside the droplet are the key to understanding the formation of dry particles with different morphology. Duffie and Marshall [1] looked at the influence of the drying and liquid feed temperatures, initial solute concentration and material properties on the resulting dry particles. They investigated aqueous solutions of salts like NaCl, KNO<sub>3</sub>, NH<sub>4</sub>NO<sub>3</sub> and others, solutions of dyes, milk, and corn syrup. An important result of their investigations was that increased drying temperatures lead to a decrease of the bulk density of the dry particulate matter due to increased particle sizes. Variations in the liquid feed temperature, in contrast, have only marginal influence on the drying result. In [2] and [3] we find the factors listed that may lead to the formation of hollow spheres in spray drying: the formation of a surface film of dry solute on the droplet (as it is the case with, e.g., aqueous NaSiO<sub>3</sub> and C<sub>12</sub>H<sub>22</sub>O<sub>11</sub> solutions), which keeps the particle size widely constant during the drying process and may finally rupture as the second drying stage starts; large drying rates as compared to the diffusive mass transport of solute in the droplet, so that hollow shells are formed; and finally the capillary effects of the solid surface, once it is formed, which can force the liquid solution towards the surface and lead to an under-pressure inside the particle. Sano and Keey [4] developed a drying model for predicting the formation of hollow particles in the drying of skim milk. These authors identified the increase of the vapor pressure at the beginning of the second drying stage as the reason for the inflation of the dried particles. Ford [5] presented a model for the drying of aqueous NaCl solution droplets, which includes the diffusional transport of the solute in the liquid, the formation of crystallization nuclei, the growth of the crystals, and the formation and growth of a solid crust on the surface of the drying droplet. In the modeling of the solid crust, a constant porosity of  $\varepsilon=0.7$  is assumed, which is typical for a dense packaging of monodispersed spheres. The result of Ford's

calculations is a morphology chart for the dry particles, which specifies the internal structure of the particles (i.e., hollow or solid spheres) as a function of initial droplet radius, drying temperature, drying air humidity, and initial salt concentration. Elapsed times to the formation of the solid crust are also given. The typical time to crust formation for NaCl solution droplets is 90ms for an initial droplet diameter of 20 $\mu$ m, initial salt concentration of 200kg/m<sup>3</sup>, a drying temperature of 80°C, and zero ambient air humidity. Brenn and coworkers [6,7] investigated this behaviour of aqueous NaCl solution droplets using an acoustic levitator. These authors identified a characteristic parameter in the non-dimensional diffusion equation for mass transport of the solute in the liquid, which may be used for characterizing the drying regimes with solid and hollow sphere formation.

It is the aim of the present work to provide an analytical solution for the concentration field in the droplet as a function of time and space during evaporation in order to predict the tendencies of the droplets to form hollow or solid dry particles. The solution is valid for cases with linear decrease of the droplet surface with time. The mathematical function developed is valid for arbitrary ratio of surface shrinkage rate to diffusion coefficient of the solute in the liquid and predicts the formation of high solute concentrations at the surface of the sphere for high drying rates accurately.

## 2. Mathematical description

We treat the problem of evaporation of a spherical droplet consisting of a binary mixture of a liquid (called the solvent) and a dissolved solid substance with very low vapor pressure (called the solute). The evaporation rate is, among others, determined by the Sherwood number imposed by the convective situation of the droplet in its host medium. The mass transfer inside the droplet, however, is considered to be controlled by diffusion and by the shrinkage of the droplet surface, i.e. we do not consider recirculating motions inside the droplet. The problem is therefore governed by the continuity and energy equations. Liquids like the ones relevant for spray drying exhibit Lewis numbers of  $O(10^2)$ , which means that the temperature relaxation goes much faster than mass diffusion. We therefore assume flat temperature profiles in the droplet and concentrate exclusively on the calculation of the concentration profiles in the liquid [8]. The droplet temperature as a function of time must of course be determined for an evaluation of the equations we develop here. In our calculations we will set it equal to the wet-bulb temperature of water at the ambient temperature and relative humidity chosen.

### 2.1 Definition of the problem

We start from the diffusion equation in spherical coordinates, assuming radially symmetric concentration fields. Note that the present derivations can be extended easily to concentration fields depending on the polar and azimuthal angles in the spherical coordinate system also. The droplet shape is assumed to be spherical all the time. The diffusion equation for the substance  $i$  in the droplet, formulated in the mass fraction  $Y_i$ , reads

$$\frac{\partial Y_i}{\partial t} = \frac{D}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial Y_i}{\partial r} \right), \quad (1)$$

where  $D$  is the binary diffusion coefficient in the liquid. In this equation, subscript  $i=1$  will be used for the solvent, and  $i=2$  for the solute. The boundary condition is that the mass fraction profile  $Y_i(r=a(t))$  at the droplet surface be determined by the equation

$$-D \frac{\partial Y_i}{\partial r} - \frac{da}{dt} Y_i = \frac{\dot{m}_i}{A \rho_l} \quad \text{at } r=a(t) \text{ for all times } t, \quad (2)$$

where  $a$  and  $A$  are the time-dependent droplet radius and surface, respectively,  $\rho_l$  is the liquid density and  $\dot{m}_i$  the mass transfer rate of substance  $i$  across the droplet surface. The latter is zero for the non-evaporating dissolved solid substance. Further to this we must satisfy the

regularity condition that the radial derivative of the mass fraction profile vanishes at the droplet center ( $\partial Y_i / \partial r = 0$  at  $r=0$ ) for all times  $t$ , and the initial condition that the mass fraction profile of substance  $i$  in the droplet at time  $t=0$  (which must also satisfy the boundary and regularity conditions, of course) be given as a function  $Y_i(t=0, r) = Y_{i0}(r)$ .

The spatial domain on which the problem is to be solved, i.e. the volume of the spherical droplet, is time-dependent. We therefore choose a similarity approach for solving the differential equation and transform it according to  $t_l = t$ ,  $\xi = r/a(t)$  into the form

$$a^2 \frac{\partial Y_i}{\partial t} = \frac{da^2}{dt} \frac{\xi}{2} \frac{\partial Y_i}{\partial \xi} + \frac{D}{\xi^2} \frac{\partial}{\partial \xi} \left( \xi^2 \frac{\partial Y_i}{\partial \xi} \right), \quad (3)$$

where we have dropped the subscript  $l$  from the time  $t_l$  [6]. The boundary, regularity, and initial conditions transform accordingly. The boundary condition (2) now reads

$$-D \frac{\partial Y_i}{\partial \xi} - \frac{\tilde{\alpha}}{2} Y_i = \frac{\dot{m}_i}{4\pi a \rho_l} \quad \text{at } \xi = l, \quad (4)$$

where we have denoted the rate of shrinkage  $da^2/dt$  by  $\tilde{\alpha}$ . The regularity condition now reads  $\partial Y_i / \partial \xi = 0$  at  $\xi = 0$ , and the initial condition becomes

$$Y_i(t = 0, \xi) = Y_{i0}(\xi). \quad (5)$$

We now render the whole problem non-dimensional with the droplet lifetime  $t_l$  defined as

$$t_l = \frac{a_0^2}{\tilde{\alpha}} \left[ \left( \frac{\bar{Y}_{20}}{(1 - \bar{Y}_{20}) \rho_{s2} / \rho_l + \bar{Y}_{20}} \right)^{2/3} - 1 \right], \quad (6)$$

and the initial droplet radius  $a_0$ . Here we denote the mean initial mass fraction of solute in the droplet by  $\bar{Y}_{20}$ , the solid density by  $\rho_{s2}$ , and the solvent density by  $\rho_l$ . The droplet lifetime as defined by equation (6) is obtained under the assumption that, at the end of the drying process, the remaining particle has the density  $\rho_{s2}$  of the compact solid substance, which is the longest possible lifetime. Using these reference quantities, we obtain the non-dimensional form of the differential equation (3) as

$$\bar{a}^2 \frac{\partial Y_i}{\partial \tau} = \alpha \frac{\xi}{2} \frac{\partial Y_i}{\partial \xi} + \frac{G}{\xi^2} \frac{\partial}{\partial \xi} \left( \xi^2 \frac{\partial Y_i}{\partial \xi} \right), \quad (7)$$

where

$$\bar{a} = \frac{a}{a_0}, \quad \tau = \frac{t}{t_l}, \quad \alpha = \tilde{\alpha} \frac{t_l}{a_0^2}, \quad G = \frac{t_l D}{a_0^2}$$

The boundary condition (4) is rewritten to get

$$-\frac{\partial Y_i}{\partial \xi} - \frac{\tilde{\alpha}}{2D} Y_i = \frac{\dot{m}_i}{4\pi a D \rho_l} \quad \text{at } \xi = l, \quad (8)$$

and the initial condition becomes

$$Y_i(\tau = 0, \xi) = Y_{i0}(\xi). \quad (9)$$

## 2.2 Analytical solution of the problem

We see that the differential equation (7) is separable in the case that the (non-dimensional) temporal derivative of the droplet surface  $\alpha$  is time-independent, i.e. if the droplet evaporates according to the so-called  $d^2$ -law. The droplet surface as a function of time then reads  $A = A_0 + 4\pi \tilde{\alpha} t$ , which is in non-dimensional form  $\bar{a}^2 = 1 + \alpha \tau$ . With the separation approach  $Y_i(\tau, \xi) = T(\tau) \cdot F(\xi)$  we obtain the ordinary differential equations

$$\frac{\bar{a}^2}{G} \frac{T'}{T} = \left( \frac{\xi}{2} \frac{\alpha}{G} + \frac{2}{\xi} \right) \frac{F'}{F} + \frac{F''}{F} := -\lambda_j \frac{\alpha}{G}, \quad (10)$$

where primes denote derivatives w.r.t. the independent variables, and  $\lambda_j$  is the separation constant. The differential equation for the function  $T(\tau)$  is readily solved to yield

$$T(\tau) = C_\tau (1 + \alpha\tau)^{-\lambda_j} . \quad (11)$$

The differential equation for the function of the non-dimensional radial coordinate  $\xi$ , however, is more difficult to solve. First we can rewrite it into the form

$$\xi^2 F'' + \left( \frac{\alpha}{2G} \xi^2 + 2 \right) \xi F' + \lambda_j \frac{\alpha}{G} \xi^2 F = 0 . \quad (12)$$

This equation is the type of differential equation 2.215 in the book by Kamke [9], which belongs to the group of hypergeometric ordinary differential equations and may be transformed by

$$F(\xi) = \mu^k \cdot \zeta(\mu) , \quad (13)$$

with  $\mu = \xi^2$ , to yield the new form

$$4\mu\zeta'' + \left( \frac{\alpha}{G}\mu + 6 + 8k \right) \zeta' + (2k + \lambda_j) \frac{\alpha}{G} \zeta = 0 . \quad (14)$$

In the transformation for the case of the present differential equation, the exponent  $k$  in (13) is to be determined as a solution of the equation

$$k(2k + 1) = 0 . \quad (15)$$

We therefore obtain two differential equations, one for  $k=0$  and one for  $k=-1/2$ , where the one for  $k=-1/2$  turns out to yield a divergent solution for  $\xi \rightarrow 0$ . Therefore we further investigate only the equation for  $k=0$ . We find its solution to read

$$\zeta(\mu) = C_1 e^{-\alpha\mu/(4G)} M\left(\frac{3}{2} - \lambda_j, \frac{3}{2}, \frac{\alpha}{4G}\mu\right) + C_2 \mu^{-1/2} e^{-\alpha\mu/(4G)} M\left(1 - \lambda_j, \frac{1}{2}, \frac{\alpha}{4G}\mu\right) . \quad (16)$$

In this solution, the function  $M(a, b, x)$  is a Kummer's function, which belongs to the group of confluent hypergeometric functions. The function is defined as [10]

$$M(a, b, x) = 1 + \frac{a}{b}x + \frac{a(a+1)}{b(b+1)} \frac{x^2}{2!} + \frac{a(a+1)(a+2)}{b(b+1)(b+2)} \frac{x^3}{3!} + \dots . \quad (17)$$

Transformation of solution (16) back into the original variables  $F$  and  $\xi$  yields the result

$$F(\xi) = C_1 e^{-\alpha\xi^2/(4G)} M\left(\frac{3}{2} - \lambda_j, \frac{3}{2}, \frac{\alpha}{4G}\xi^2\right) + C_2 \frac{1}{\xi} e^{-\alpha\xi^2/(4G)} M\left(1 - \lambda_j, \frac{1}{2}, \frac{\alpha}{4G}\xi^2\right) . \quad (18)$$

An inspection of Eq. (18) reveals that the function with the coefficient  $C_2$  diverges as  $\xi \rightarrow 0$ , i.e. at the droplet center. This part of the solution must therefore be excluded by setting  $C_2=0$ . Making use of the Kummer transformation

$$e^z M(j-i, j, -z) = M(i, j, z) , \quad (19)$$

we obtain as the general solution of the differential equation (12)

$$F(\xi) = C_\xi M\left(\lambda_j, \frac{3}{2}, -\frac{\alpha}{4G}\xi^2\right) . \quad (20)$$

We can therefore compose an eigensolution of the diffusion equation (7), according to our separation approach, in the form

$$Y_{i,j}(\tau, \xi) = C_j (1 + \alpha\tau)^{-\lambda_j} \cdot M\left(\lambda_j, \frac{3}{2}, -\frac{\alpha}{4G}\xi^2\right) , \quad (21)$$

and obtain the general solution of the equation as the linear combination

$$Y_i(\tau, \xi) = \sum_j C_j (1 + \alpha\tau)^{-\lambda_j} \cdot M\left(\lambda_j, \frac{3}{2}, -\frac{\alpha}{4G}\xi^2\right) \quad (22)$$

of all these eigensolutions. An inspection of equation (22) reveals that the functions satisfy the regularity condition at the droplet center. The coefficients  $C_j$  in the series expansion are determined by satisfying the initial condition (9). The boundary condition (8) must be

satisfied by all eigensolutions separately, i.e. by all Kummer's functions in (22). This boundary condition is particularly simple for the non-evaporating solute  $i=2$ , since it is homogeneous in this case. Formulating this condition by substituting (21) into the boundary condition (8), we obtain for  $\dot{m}_2=0$  the requirement that

$$M'\left(\lambda_j, \frac{3}{2}, x\right) - M\left(\lambda_j, \frac{3}{2}, x\right) = 0 \quad \text{at } \xi=1. \quad (23)$$

In this equation, the prime denotes the derivative of the function  $M$  w.r.t. the third argument  $x=-\alpha\xi^2/(4G)$ . Making use of the identity

$$(j-i)M(i, j+1, x) = j[M(i, j, x) - M'(i, j, x)], \quad (24)$$

we see that Eq. (23) can be satisfied by letting

$$\lambda_0 = \frac{3}{2} \quad \text{or for} \quad M\left(\lambda_j, \frac{5}{2}, -\frac{\alpha}{4G}\right) = 0 \quad (25)$$

I.e., we obtain solutions of our problem by choosing the eigenvalues  $\lambda_j$  as  $3/2$  (for  $j=0$ ) or such that the value of the quantity  $-\alpha/(4G)$  becomes the  $j$ th zero ( $j>0$ ) of the function  $M$  in (25). These zeros are computed following a procedure given by Abramowitz and Stegun [10]. The expansion coefficients  $C_j$  in (22) must be determined such that the expansion (22) represents the initial distribution of the mass fraction  $Y_{i0}(\xi)$  in the droplet for  $\tau=0$ . For doing this, we need an orthogonality property of the eigenfunctions of the problem. Inspection of the ordinary differential equation (12) reveals that the quantity  $-\alpha\lambda_j/G$  is an eigenvalue of the differential operator

$$\frac{\partial^2}{\partial \xi^2} + \left( \frac{\alpha}{2G} \xi + \frac{2}{\xi} \right) \frac{\partial}{\partial \xi}. \quad (26)$$

This operator is not symmetric w.r.t. the formation of scalar products in Hilbert spaces, so that we cannot expect the eigenfunctions satisfying the differential equation (12) to be orthogonal, nor can we expect that the system of functions (20) be complete nor that the eigenvalues be real. However, trying to convert the operator into a symmetrical form by multiplying the differential equation (12) with a function  $q$ , i.e., requiring

$$qF'' + q\left(\frac{\alpha}{2G}\xi + \frac{2}{\xi}\right)F' = \frac{\partial}{\partial \xi}\left(q\frac{\partial F}{\partial \xi}\right) = -\lambda_j \frac{\alpha}{G}qF, \quad (27)$$

we find that we can make the differential operator symmetrical. This requires appropriate determination of the function  $q$ , for which we obtain a differential equation by differentiating the central part of (27) and comparing coefficients with the left-hand part [11]. The solution of this differential equation is

$$q(\xi) = \xi^2 e^{\alpha\xi^2/(4G)}. \quad (28)$$

Since this procedure worked just with a multiplication of the original ordinary differential equation (12) with a function  $q$ , the solution (20) remains valid, and, in order to obtain the coefficients  $C_j$  in the expansion (22), we can now make use of the orthogonality of two functions  $M$  when multiplied by the function  $q$  given by (28). This means that we can now calculate the coefficients  $C_j$  such that the expansion (22) represents the initial distribution  $Y_{20}(\xi)$  of the mass fraction of the solute in the droplet liquid. The equation reads

$$C_j = \frac{\int_{\xi=0}^1 q Y_{20}(\xi) M(\lambda_j, 3/2, -\alpha\xi^2/(4G)) d\xi}{\int_{\xi=0}^1 q M(\lambda_j, 3/2, -\alpha\xi^2/(4G)) \cdot M(\lambda_j, 3/2, -\alpha\xi^2/(4G)) d\xi}. \quad (29)$$

With this equation, the problem of diffusional mass transport of the solute inside the droplet is solved. With the field of the mass fraction  $Y_2(\tau, \xi)$  of the solute known, we can easily calculate the field  $Y_1$  of the solvent as  $Y_1(\tau, \xi) = 1 - Y_2(\tau, \xi)$ .

### 3. Evaluation of the equations and example calculations

We evaluate the equations found in the preceding section for the special case of the parabolic initial distribution

$$Y_{20} = \gamma \xi^2 + \delta, \quad (30)$$

which is the simplest initial mass fraction distribution that satisfies both the boundary condition at  $\xi=1$  and the regularity condition at the droplet center. The coefficients in (30) are determined by the boundary condition and by the requirement that the integral of the distribution  $Y_{20}(\xi)$  over the droplet volume must yield  $\bar{Y}_{20}$  times the initial droplet volume. The coefficients read

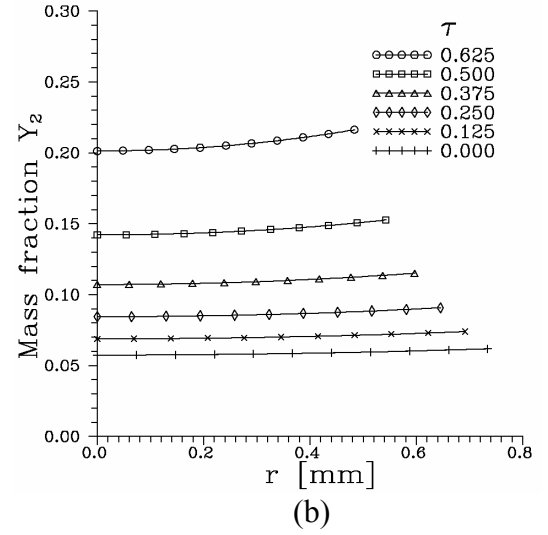
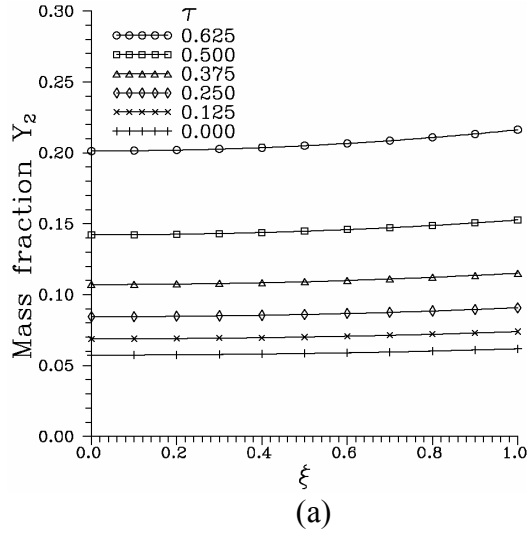
$$\begin{aligned} \gamma &= \frac{5\bar{Y}_{20}/3}{1 - 5(1 + 4D/\tilde{\alpha})/3} \quad \text{and} \\ \delta &= \frac{-5(1 + 4D/\tilde{\alpha})/3}{1 - 5(1 + 4D/\tilde{\alpha})/3} \bar{Y}_{20}. \end{aligned} \quad (31)$$

In these equations,  $\bar{Y}_{20}$  is the overall mean mass fraction of solute in the droplet at  $\tau=0$  (which we assume to be known as a part of the initial conditions for the drying process). Having chosen a set of input data  $a_0$ ,  $\bar{Y}_{20}$ ,  $D$  and  $\tilde{\alpha}$ , we compute the eigenvalues  $\lambda_j$  from (25), and the expansion coefficients  $C_j$  from (29), and then compute the values of the function (22) at various times  $0 \leq \tau \leq 1$  and non-dimensional radial positions  $0 \leq \xi \leq 1$ , varying the ratio  $\tilde{\alpha}/D$  of rate of droplet shrinkage to diffusion coefficient of the solute in the solvent. For the rate of droplet shrinkage we take typical values observed in our levitator experiments [6]. The diffusion coefficient we take as concentration-independent, which is a valid assumption for a wide range of concentrations. The rate of shrinkage may be associated with the Sherwood number of mass transfer across the droplet surface and with the other relevant drying parameters according to the equation

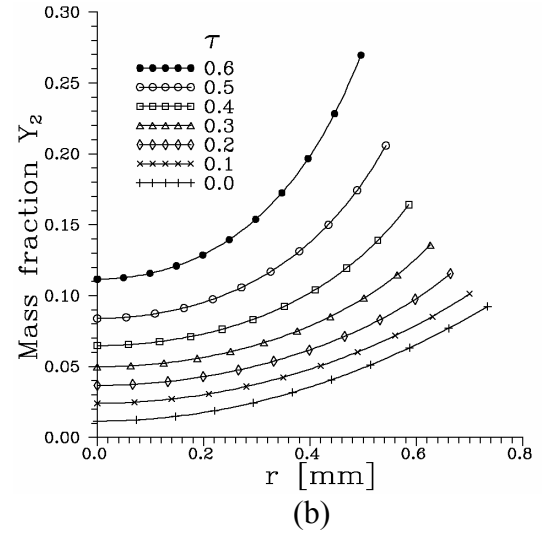
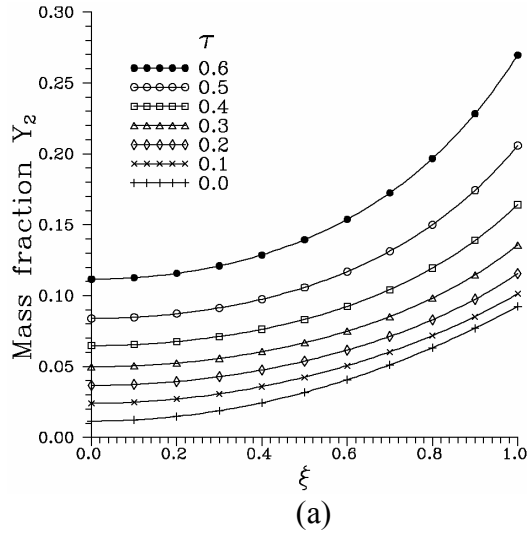
$$\alpha = \frac{\rho_g}{\rho_l} \frac{D_g}{D} G Sh^* \ln(1 + B_M), \quad (32)$$

where  $\rho_g$  is a mean gas density around the droplet,  $\rho_l$  is the liquid solvent density,  $D_g$  the diffusion coefficient of the evaporated solvent in the ambient air,  $Sh^*$  a modified Sherwood number as introduced in [12], and  $B_M$  the Spalding mass transfer number. This equation quantifies the change of the Sherwood number with time to be realized according to the temporal change of  $B_M$  due to the increasing concentration of solute near the surface of the droplet during the drying process. A similar equation was developed in [7]. In that paper, however, the convective mass transfer of liquid vapor from the droplet surface was caused by an acoustic streaming flow. The Sherwood number had therefore to be modeled accordingly.

As an example we calculate the drying of droplets of an aqueous sodium chloride solution at various rates of droplet shrinkage  $\alpha$ . We take  $\alpha$  as constant, as assumed in our theory, choosing different ratios of  $\alpha/G$  far below and far above 1. The following two figures show the computational results achieved by evaluating Eq. (22) together with (25) and (29), using the above procedure. It turned out that a maximum number of 10 terms in the expansion (22) is sufficient to have an accurate description of the mass fraction profiles. The data shown in Fig. 1 belong to the case for a low evaporation rate and show flat concentration profiles, as expected, since the diffusive transport of solute in the droplet liquid has enough time in this case to redistribute the solute from the surface of the droplet towards its center while the solvent evaporates. The profiles in Fig. 2, in contrast, are found for a relatively high evapora-



**Fig. 1** (a) Evolution of the mass fraction  $Y_2$  of solute in non-dimensional coordinates, and (b) – in the dimensional radial coordinate showing the shrinkage of the droplet with time. Data:  $\rho_l=1000\text{kg/m}^3$ ,  $\rho_{s2}=2165\text{kg/m}^3$ ,  $\alpha'=-0.46\cdot 10^{-9}\text{m}^2/\text{s}$ ,  $D=1.61\cdot 10^{-9}\text{m}^2/\text{s}$ ,  $\bar{Y}_{20}=0.06$ ,  $Y_{sat,2}=0.2629$ ,  $a_0=0.734\text{mm}$ . Diffusion coefficient and saturation mass fraction correspond to a droplet wet bulb temperature of  $8^\circ\text{C}$ , i.e. to an ambient air temperature of  $20^\circ\text{C}$  and a relative humidity of 4.7%.



**Fig. 2** (a) Evolution of the mass fraction  $Y_2$  of solute in non-dimensional coordinates, and (b) – in the dimensional radial coordinate showing the shrinkage of the droplet with time. Data:  $\rho_l=1000\text{kg/m}^3$ ,  $\rho_{s2}=2165\text{kg/m}^3$ ,  $\alpha'=-5.635\cdot 10^{-9}\text{m}^2/\text{s}$ ,  $D=1.61\cdot 10^{-9}\text{m}^2/\text{s}$ ,  $\bar{Y}_{20}=0.06$ ,  $Y_{sat,2}=0.2629$ ,  $a_0=0.734\text{mm}$ . Diffusion coefficient and saturation mass fraction correspond to a droplet wet bulb temperature of  $8^\circ\text{C}$ , i.e. to an ambient air temperature of  $20^\circ\text{C}$  and a relative humidity of 4.7%.

tion rate of the solvent. Consequently, larger gradients of the mass fraction of the solute are seen. The curves in the figures are profiles for different non-dimensional times, starting with the initial states  $\tau=0$ . The uppermost curves in the figures show the situation where the saturation concentration  $Y_{sat,2}$  is reached at the surface of the droplet for the first time. The profiles show a clear tendency of the drying droplets to turn into hollow spherical shells for the high evaporation rate, while the low evaporation rate clearly leaves enough time for producing solid spheres. The analytical solution of the diffusion equation is seen to yield physically plausible results.

#### 4. Conclusions

An analytical solution of the diffusion equation on a spherical domain with moving boundary is developed in order to quantify the diffusive transport of the components of a binary mixture inside the droplet during evaporation of the solvent. The differential equation governing the problem is solved using a separation approach for the special case of time-independent rate of shrinkage of the droplet. The dependency of the solute concentration on the radial coordinate is found to be given by a Kummer's function, which belongs to the group of confluent hypergeometric functions. The solution is valid for arbitrary values of the ratio of rate of droplet shrinkage to diffusion coefficient in the droplet liquid. An investigation of the analytical function for typical drying situations reveals the expected behavior, i.e. the formation of large concentration gradients in the droplet at high evaporation rates, and flat concentration profiles for slow drying. Times elapsed until the state of saturation is reached are quantified. Other phenomena in drying like precipitation and crust formation are not included in the calculations.

#### Acknowledgements

The present work was carried out in parallel with the EU research project ENK6-CT2000-00101. The financial support from the EU commission is gratefully acknowledged. The author gratefully acknowledges many fruitful discussions with his colleagues Prof. Dr. W. Borchers, Institute of Mathematics (University of Erlangen-Nürnberg, Germany), Prof. Dr. A.L. Yarin, Faculty of Mechanical Engineering (Technion – Israel Institute of Technology, Haifa, Israel), and Dr. M. Göz, Institute of Particle Technology and Environmental Process Engineering (University of Halle-Wittenberg, Germany). Without their advice, this work would not have been completed.

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