Far-field penetration of mushroom-like sprays

Roisman I. V, Tropea C.

Technische Universität Darsmtadt, Darmstadt, Germany

In the present theoretical work the propagation of a fuel spray at distances much longer than the break-up length is studied. Such propagation is followed by the creation of a special front-edge region in the spray. The flow in this region leads to the characteristic mushroom-like shape. The momentum balance of the front edge of the spray is considered yielding the equation of propagation of the spray. The predicted penetration length of the sprays agrees well with the experimental data.

1. Introduction

Experimental investigations of fuel sprays, as well as its modelling is extremely challenging problem. The spray, especially in the neighbourhood of the nozzle, is very dense and that makes the use of the conventional optical measurement techniques (such as the phase-Doppler technique or PIV) very difficult. This is the reason why the spray penetration and its cone-angle, which can be obtained using photographic techniques, are among the most frequently reported parameters in fuel-spray research. Such a technique was used in one of the first experimental studies (Mille & Beardsley 1926, [1]) about the effect of ambient pressure (or in fact the density of the ambient gas) on the penetration depth of the engine spray. The images show that the spray develops a mushroom-like shape. This shape is observed in most of the experimental studies of spray injection *e.g* in the images presented in recent studies [2]-[4].

In Fig. 1 the shape of the spray obtained in [5] at various time instants is shown for two

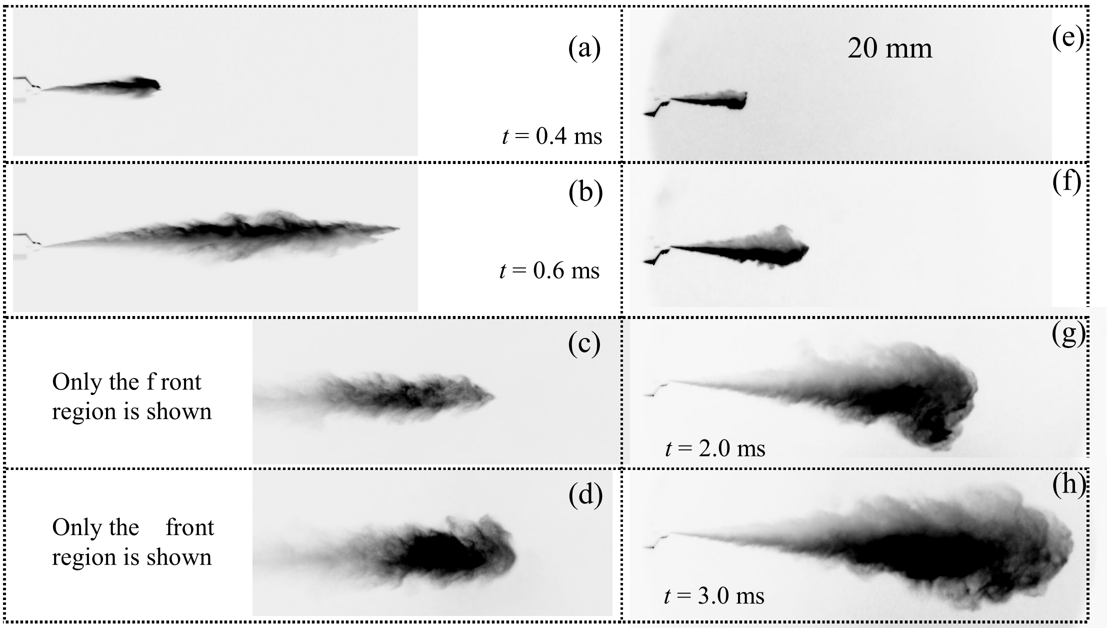


Fig. 1: Shape of the spray at different instants after injection. The ambient pressure is p_a = 1 bar (images on the left) and p_a = 25 bar (images on the right). The time instants are: (a) and (e) 0.4 ms; (b) and (f) 0.6 ms; (c) and (g) 2.0 ms; (d) and (h) 3.0 ms.

different ambient pressures: p_a = 1 bar and p_a = 25 bar. All other parameters of injection are the same. It can be seen that the mushroom-like shape is more pronounced for the case of higher ambient pressure, indicating that the air drag at the front edge determines the shape of the cloud of the particles and its penetration.

Similar, mushroom-like shapes are observed in the galaxy "worms" of the length of order of 1000 light years [6], and in the experiments associated with the penetration mechanics where metal rods or shaped-charge jets move with the velocity of order of 10^3 m/s. Hydrodynamic models of the type considered in [7] - [9] allow one to successfully predict the penetration of the shaped-charge jets and eroding projectiles.

There are several models, empirical or numerical, of spray penetration. In a recent study [10] the penetration of a spray was calculated by solving the cross-sectionally averaged equations of the flow. The conditions at the front edge of the spray have not yet been considered for dense sprays.

It is known that spray penetration can be subdivided into two parts. The first short phase (just after injection instant), when the injected fuel jet breaks up, and the second, steady-state phase, which is the main subject of the present study.

The proposed hydrodynamic model of penetration of a dense fuel spray in the present work takes into account the inertia of the ambient gas and the mushroom-like shape of the cloud of the liquid drops. It is assumed that the evaporation of droplets in the spray does not influence the total momentum of the penetrating spray and its effect is not considered in the present study. This assumption is supported by the results of calculations of the spray penetration length [10]. These results are only slightly influenced by taking into account the evaporation.

2. Model

Consider the spray propagating in the ambient air. In the present model the spray is subdivided into two main regions (see Fig. 2). The region ① is far from the front edge of the spray. The flow here is assumed to be steady, the pressure gradient is negligibly small. At the frontal edge of the spray the droplets are collected in the mushroom-like region ②.

The drops in the spray are very small. The penetration of a single 5 μm drop with an initial velocity of 200 m/s into ambient air is approximately 5 mm. Therefore, we can approximate

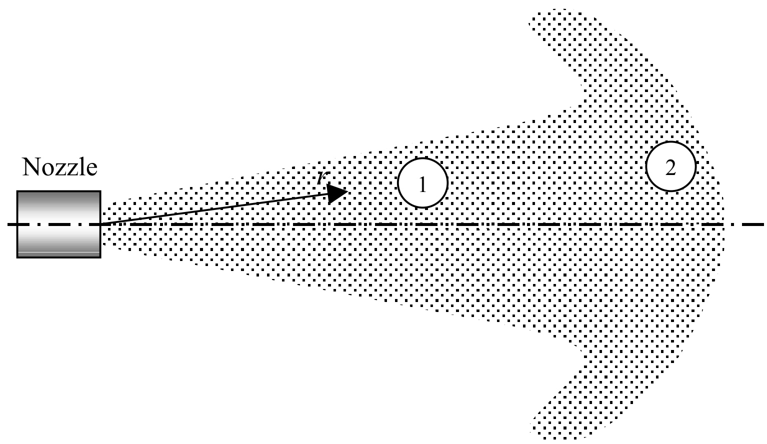


Fig. 2. Sketch of the propagating spray. ① corresponds to the region of the steady radial expansion, ② is the spray collected in the frontal mushroom-like region.

the velocity of the surrounding gas by the local average velocity of the drops in the spray. This assumption is supported by the results of the study [11] showing that the values of the spray penetration predicted by the model is virtually not influenced by the drag model.

Steady flow in the region ①

Consider the steady-state flow in the spray in the region ① far behind the front of the spray (the injection is assumed to be steady). Consider a spherical coordinate system with the origin at the nozzle region. We also assume that at the distances much larger than the nozzle diameter the spray propagates radially with the velocity $u(r)$. This strong assumption is confirmed by wide variety of experimental studies of the spray propagation showing an almost conical shape of the spray with the constant half-angle in the steady region ①.

The total mass conservation of the liquid phase and the momentum balance of the mixture in the radial direction lead to (assuming no evaporation):

$$\frac{\partial}{\partial r}[r^2\,\gamma(r)\,u(r)] = 0\,, \tag{1a}$$

$$\frac{\partial}{\partial r}[\rho\, r^2\,\gamma(r)\,u(r)^2 + \rho_a r^2\,(1-\gamma(r))\,u(r)^2] = 0 \tag{1b}$$

where γ is the volume fraction of the liquid phase, $u(r)$ is the radial velocity of the mixture, ρ and ρ_a are the densities of the liquid and air.

The solution of the above equations is:

$$\gamma = \frac{C_1}{2r^2}A(r)\,,\qquad u = \frac{C_1 C_2 \Delta\rho}{A(r)}\,,\qquad A(r) = C_1\Delta\rho + \sqrt{C_1^2\Delta\rho^2 + 4r^2\rho_a} \tag{2a,b,c}$$

where $\Delta\rho = \rho - \rho_a$, and the integration constants C_1 and C_2 are given by

$$C_1 = \frac{R_0}{\sqrt{\rho}}\,,\qquad C_2 = 2U_0\left(1+\frac{\rho_a}{\Delta\rho}\right). \tag{3a,b}$$

The radius R_0 corresponds to the coordinate of the beginning of the break-up of the injected jet. At this position the velocity is U_0 and $\gamma = 1$. It depends on the geometry of the nozzle, density of the gas and the fuel, and the intensity of the deformations of the jet.

Initial conditions, atomization of the jet.

Although the break-up region of the injected jet is the region very difficult to investigate experimentally, it is known that the cavitation phenomenon influences the break-up process significantly. The cavitation leads to the emergence of the number of voids randomly distributed in the jet. Therefore, the atomization of the jet can be described using the "chaotic disintegration" approach of [12]. The volume fraction $\gamma \sim r^{-2}$ of the liquid phase reduces with the distance from the nozzle, whereas the volume fraction $1-\gamma$ of the voids increases. Consequently, their size and the number density grow and they begin to merge leading to the fragmentation of the jet. The jet in this intermediate region can be described as a liquid infinite cluster occuping the relative volume γ. In the terms of the percolation theory [13] the probability that an elementary particle of size a (the smallest diameter of the drops which will be created after the break up) occupies a site in the element of liquid volume is γ, whereas the probability that it belongs to a lacunae is $(1-\gamma)$. At some critical probability (the percolation threshold, [8], [12], [13], [14]), corresponding to $\gamma^* = 0.311$ in the three-dimensional case the "infinite" cluster is broken and the distribution of finite clusters (droplets of various diameters).

The volume fraction γ of the liquid phase can be approximated in the case $\rho_a << \rho$ in the form

$$\gamma \approx \frac{R_0^2}{r^2} \quad \text{if } \rho_a << \rho, \tag{4}$$

the breakup length can be therefore estimated as

$$l^* = r_{|\gamma=\gamma^*} \approx 1.79\, R_0 \tag{5}$$

Propagation of the spray tip, analysis of the region ②.

The above analysis is not valid at the front of the spray where the volume fraction of the liquid phase jumps from γ to 0. The resistance of the air leads to the collection of the spray in the front region and produces the mushroom-like shape of the spray. Denote U_T the velocity of the tip of the spray which differs from the radial velocity u of the mixture. The pressure in the air at the tip of the spray can be estimated using the Bernoulli equation as $\rho_a U_T^2 / 2$. The momentum equation of the front edge of the spray expresses the balance of the inertia of the liquid and the gas phases of the spray entering the front region with the relative velocity u - U_T and the pressure of the gas at the front region just outside the spray:

$$[\rho\gamma + \rho_a(1-\gamma)](u - U_T)^2 = \frac{\rho_a U_T^2}{2} \tag{6}$$

The solution of equation (4) for $U_T < u$ is:

$$U_T = u\left[1 + \sqrt{\frac{\rho_a}{2(\rho_a + \Delta\rho\gamma)}}\right]^{-1} \tag{7}$$

Now the ordinary differential equation for the temporal propagation of the spray tip can be written in the form

$$\frac{\mathrm{d}R_T}{\mathrm{d}t} = U_T \tag{8}$$

Here R_T is the penetration length .

Solution

The solution can be written in dimensionless form taking R_0 as a length scale and U_0 as a velocity scale. Denoting

$$\mu = \frac{\rho_a}{\rho - \rho_a}, \qquad 1 + \frac{\mu}{\gamma_0} = \eta^2 \tag{9}$$

yields:

$$\gamma = \frac{1}{2\eta\bar{r}^2}\left[1 + \sqrt{1 + 4\eta\bar{r}^2\mu}\right], \qquad \bar{u} = \frac{2\eta^2}{1 + \sqrt{1 + 4\eta\bar{r}^2\mu}} \tag{10a,b}$$

$$\bar{U}_T = \frac{\bar{u}}{1 + \sqrt{\dfrac{\mu}{2(\mu + \gamma)}}} \,. \tag{10c}$$

Here and below the overbared variables are dimensionless.

The solution of equation (8) for the penetration length after break up can be written in the following form:

$$\bar{t} = \int_{\bar{R}_0}^{\bar{R}_T} \frac{1}{\bar{u}(\bar{r})}\left[1 + \sqrt{\frac{\mu}{2(\mu + \gamma(\bar{r}))}}\right] \mathrm{d}\bar{r} \tag{11}$$

At the relatively long distance ($\mu^{1/2}\eta^{1/2} r >> 1$) the approximated solution for the penetration length can be written in the form:

$$\bar{R}_T \approx a + \sqrt{2}\sqrt{2 - \sqrt{2}}\, \mu^{-1/4}\eta^{3/4}(t + \tau)^{1/2} \tag{12}$$

where a and τ are some constants. The above square root temporal dependence of the penetration length at long distances was reported in the experimental studies [18].

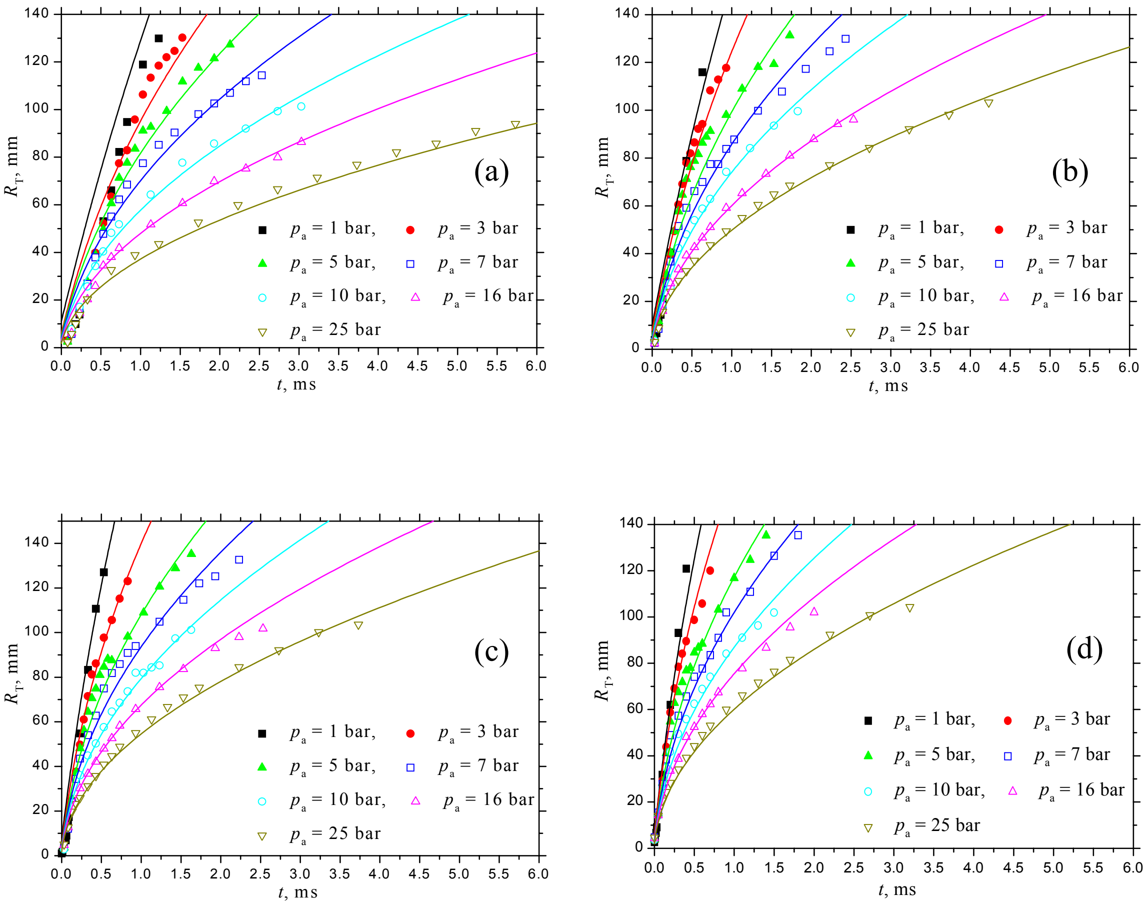
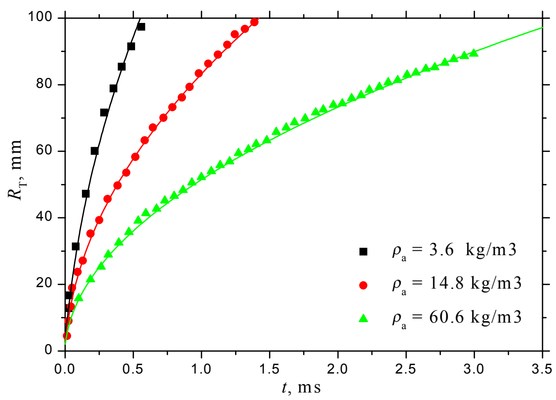


Fig. 3. Influence of the ambient pressure on the spray penetration. The injection pressures are: (a) p_i = 300 bar, (b) p_i = 700 bar, (c) p_i = 1000 bar, (d) p_i = 1350 bar. The continuous lines correspond to the proposed model. The experimental data is of Kuß [4].

3. Results and discussion

The main source of the experimental data used for validation of the model is the results [4] obtained in our laboratory (SLA, TU-Darmstadt). In these experiments the silicone oil with the parameters similar to that of the diesel fuel (density $\rho = 817$ kg/m^3, surface tension σ = 26.31 mN/m, viscosity: $\mu = 2.867\ 10^{-3}$ kg/m·s) was injected into the pressure chamber. The nozzle inlet diameter is 0.190 mm. The injection pressure and the ambient pressure in the chamber were varied in the experiments.

The results of comparison of the penetration length R_T predicted by the proposed model (eq. 11) with the experimental data is shown in Fig. 3. The agreement is rather good. Similar results are obtained comparing the model with the experimental data from literature (see Figs. 4-6). One fitted parameter, R_0, is the downstream distance at which the jet breaks up. The experimental data for the spray penetration [15] in Fig. 4 is taken from the graphs in paper [10].



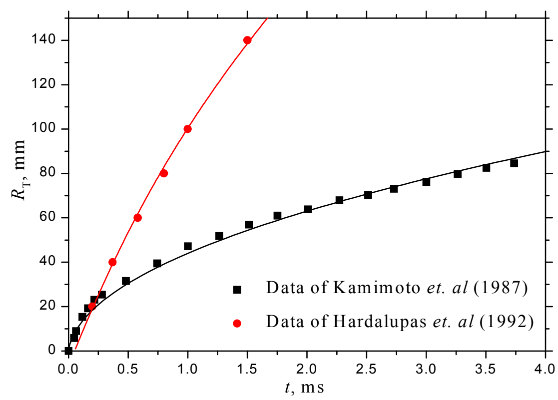


Fig. 4. Influence of the density of the ambient gas on the spray penetration. The data are of Naber *et. al* [15]. The continuous lines correspond to the proposed model.

Fig. 5. Spray penetration data in comparison with the proposed model. The data are from Kamimoto *et al*. [16] and Hardalupas *et al.* [17].

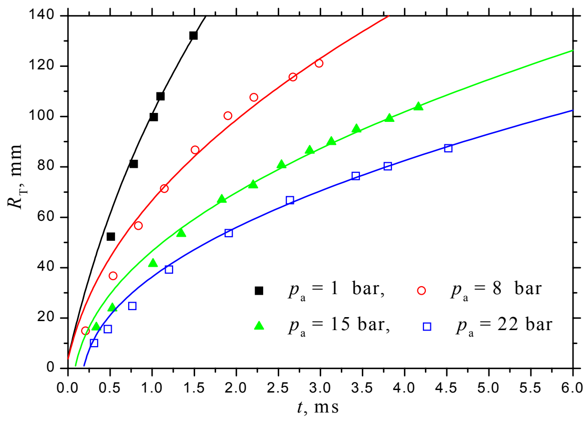
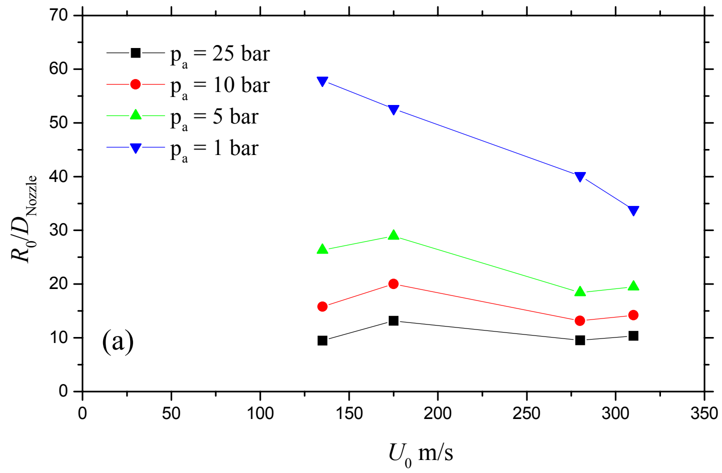


Fig. 6. Influence of the density of the ambient gas on the spray penetration. The experimental data is of Miller&Beardsley [1]. The continuous lines correspond to the proposed model.



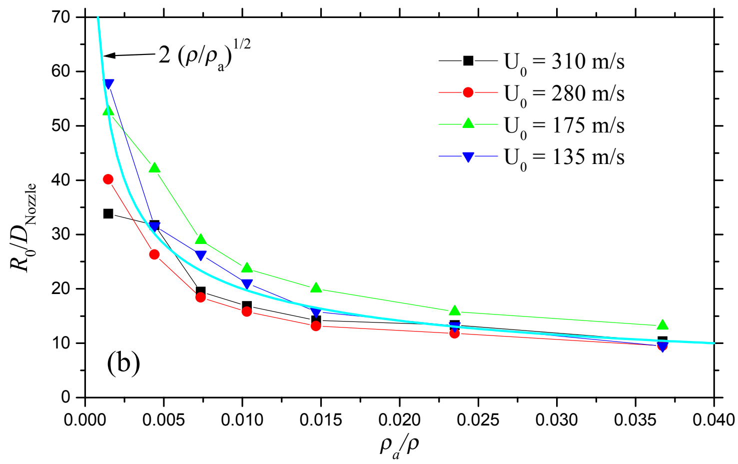


Fig. 7. Radius R_0 normalize to the inlet diameter of the nozzle: (a) as a function of injection velocity U_0; (b) as a function of the relative density of the ambient air. The data are from Kuß [4].

In Fig. 7 R_0 obtained by fitting of the experimental data [4] using the expression (11), is shown as a function of the injection velocity and the density of the ambient gas. Fig. 7a shows that at low ambient pressure (1 bar) the radius R_0 decreases significantly with the injection velocity U_0. This result is in qualitative agreement with the results of the study [18] of the break-up length (which is proportional to R_0) indicating that the experiments [4] (shown in Fig. 7) can be associated with the "spray region" of the break-up.

At higher pressures, as shown in Fig. 7a, the dependence of R_0 on the injection velocity U_0 is not so significant. This result is also in the agreement with the conclusions of the study [18] that at higher injection velocities the break-up length approaches some constant value. The break-up length reaches these constant values at a smaller velocity if the ambient pressure is higher. At these higher ambient pressures the parameter R_0 depends on the gas density (see Fig. 7b) and follows the square root law obtained in [18] for the break-up length.

Conclusions

The propagation of a fuel spray is an extremely complicated phenomenon, including the jet injection and break-up, atomization (primary and secondary), very aggressive turbulent flow in the air and in the liquid phases, interaction of droplets in the spray and their evaporation.

In the proposed model most of the details of the transport in the diesel spray are neglected. Only two main factors are accounted for:

- the inertia of the liquid/air mixture in the steady conical region of the spray;
- formation of the mushroom-like shape of the spray due to the air drag at the front region of the spray.

The proposed model predicts successfully the integral parameters of the spray (spray tip penetration) and the distribution of the volume fraction of the liquid phase in the spray.

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